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Optimal Cooperative Line-of-Sight Guidance for Defending a Guided Missile

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Abstract: This correspondence proposes an optimal cooperative guidance law for protecting a target from a guided missile. The linearized three-body kinematics using the line-of-sight (LOS) triangle concept is formulated, and a new concept called error distance is introduced. A generalized linear quadratic optimization problem is formulated in minimizing weighted energy consumption while regulating the error distance. The analytic guidance command is derived by solving the optimization problem formulated. The main feature of the proposed guidance law lies in that it helps reduce the maneuver capability demand of the defender. Extensive numerical simulations are carried out to demonstrate the effectiveness of the proposed solution.

Keywords: active aircraft protection; cooperative guidance; line-of-sight guidance; optimal control

1. Introduction

Due to the velocity and maneuverability advantage, missiles pose a severe threat to aircraft in modern warfare. To improve the survival probability of aircraft, numerous defensive methods have been reported and can be generally divided into two categories: passive defense and active defense. Passive defense mainly depends on performing evasive maneuvers or releasing deceptive decoys. However, with the development of the attacking missile's sensing and maneuver capabilities, the passive defensive method is insufficient to ensure the safety of the aircraft. Hence, active defense has been proposed to further enhance the survivability of the aircraft [1–3]. The primary active defense strategy utilized by the aircraft is launching a defensive missile to intercept the incoming missile, which is known as the target–missile–defender three-body problem [4,5].

By considering the defense missile and attacking missile engagement separately from the target, the three-body problem can be simplified into two one-on-one engagements: attacking missile to target engagement and defender to attacking missile engagement. From this standpoint, the defense problem can be defined as an interception problem for the defender or an evasive guidance problem for the target aircraft [6,7]. Within this framework, existing guidance laws developed from optimal control theory [8,9], e.g., proportional navigation guidance (PNG) and its variants [10–13], can be directly applied. However, a separate one-on-one design philosophy cannot fully exploit the synthetic effect of the cooperation between the aircraft and the defender. For this reason, cooperative guidance for active aircraft protection has attracted extensive interest in recent years [14,15]. For example, the authors in [16] developed a linear cooperative guidance algorithm for the defender to pursue the attacker and the target aircraft to evade the attacker simultaneously in minimizing the control effort. This work was later extended to a nonlinear guidance law using sliding mode control (SMC) in [17,18]. With no information on the guidance law implemented by the attacking missile, an optimal cooperative guidance law was proposed and sufficient conditions for the existence of the optimal solution were also derived in [19]. Different from energy minimization, the authors in [20] presented a differential game



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guidance law that strove to maximize the terminal separation between the target aircraft and the attacker at the time instant where the attacker was intercepted by the defender.

Recently, the target–missile–defender three-body problem was also recast into a three-point guidance problem, i.e., the defender can protect the target aircraft as long as it can move along the LOS from the target to the attacker. This geometric rule was first leveraged in [21–23] for active aircraft protection by manipulating two LOS rates associated with the three players. However, this algorithm assumed no cooperation between the target and the defender. For this reason, the authors in [24] developed a cooperative command to LOS (CLOS) guidance law by minimizing the defender-to-attacker maneuverability ratio. The potential of CLOS guidance in active aircraft protection was later analyzed in [25] by comparing PNG and pursuit guidance in a game theory framework. Later in [26,27], a nonlinear cooperative CLOS guidance law using SMC was proposed, and the lateral acceleration command was optimally allocated to the defender–target team. However, these guidance strategies did not consider any control effort minimization in deriving the guidance commands.

This paper proposes an optimal cooperative defense guidance law both in implicit form and explicit form for a defense system that comprises a target and a launched missile. We first formulate a linear three-body kinematics model by the three-point guidance geometry and introduce a new concept called error distance. A generalized linear quadratic optimal control problem with arbitrary weighting functions for both implicit and explicit cooperative defense guidance problem is then formulated, and the analytical solution is derived with optimal control theory. The potential of the proposed guidance law is that we can further reduce the defender-to-attacker maneuverability ratio by placing different energy penalty weights on the defender and the attacker. The benefits of the guidance law developed are also compared with existing solutions with numerical simulations. To the best of our knowledge, no similar results on optimal cooperative guidance law have been reported in the literature.

2. Three-Body Kinematics Equations and Linearization

2.1. Nonlinear Relative Kinematics

The planar three-body engagement is shown in Figure 1. The defending missile, denoted by D , is launched from the target T . The objective is to protect the target away from the attacking missile, which is denoted by M . The relative ranges and LOS angles between the three vehicles are, respectively, represented by r_{ij} and λ_{ij} , where $i, j \in \{T, D, M\}$ and $i \neq j$. The notation γ_i denotes the flight path angle, and V_i stands for the velocity. Since the moving speed is generally slow-varying for aerodynamically-controlled vehicles, we assume V_i is constant, and only its heading direction can be adjusted by lateral acceleration u_i .

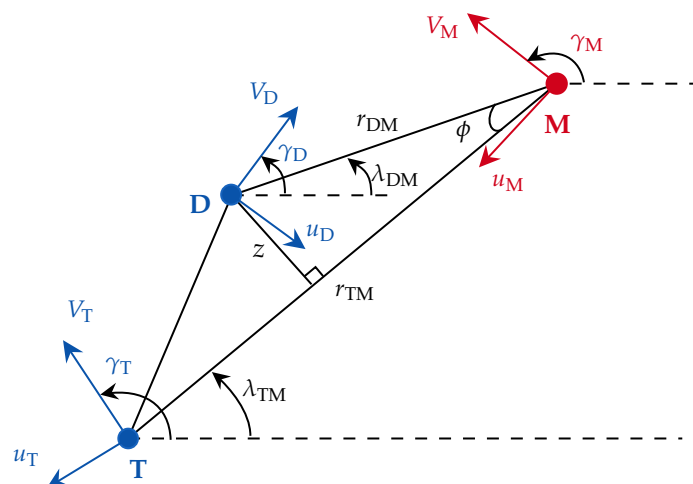


Figure 1. Three-body engagement geometry.

According to the geometry, the range rates and LOS rates can be readily obtained as

$$\dot{r}_{DM} = V_M \cos(\gamma_M - \lambda_{DM}) - V_D \cos(\gamma_D - \lambda_{DM}) \tag{1}$$

$$\dot{r}_{TM} = V_M \cos(\gamma_M - \lambda_{TM}) - V_T \cos(\gamma_T - \lambda_{TM}) \tag{2}$$

$$r_{DM} \dot{\lambda}_{DM} = V_M \sin(\gamma_M - \lambda_{DM}) - V_D \sin(\gamma_D - \lambda_{DM}) \tag{3}$$

$$r_{TM} \dot{\lambda}_{TM} = V_M \sin(\gamma_M - \lambda_{TM}) - V_T \sin(\gamma_T - \lambda_{TM}) \tag{4}$$

Differentiating Equations (3) and (4) with respect to time yields

$$\ddot{\lambda}_{DM} = -2 \frac{\dot{r}_{DM}}{r_{DM}} + \frac{\cos(\gamma_M - \lambda_{DM})}{r_{DM}} u_M - \frac{\cos(\gamma_D - \lambda_{DM})}{r_{DM}} u_D \tag{5}$$

$$\ddot{\lambda}_{TM} = -2 \frac{\dot{r}_{TM}}{r_{TM}} + \frac{\cos(\gamma_M - \lambda_{TM})}{r_{TM}} u_M - \frac{\cos(\gamma_T - \lambda_{TM})}{r_{TM}} u_T \tag{6}$$

where $u_i (i \in \{T, D, M\})$ denotes the lateral acceleration.

Let t_{fij} represent the final time with $i, j \in \{T, D, M\}$ and $i \neq j$. The remaining flight time, or the so-called time-to-go, can be approximately calculated as

$$t_{gij} = t_{fij} - t \approx -\frac{r_{ij}}{\dot{r}_{ij}} \tag{7}$$

2.2. Linear Error Distance Dynamics

In this paper, we describe the engagement geometry with the triangle connected by three lines of sight: LOS_{TD} , LOS_{DM} , and LOS_{TM} . The perpendicular distance from defender to LOS_{TM} , denoted by z , is defined as the error distance. Define $\phi = \lambda_{TM} - \lambda_{DM}$, and it can be considered as a small angle near the collision course. Then, the error distance can be expressed as

$$z \approx r_{DM} \phi \tag{8}$$

which subsequently gives the dynamics of the error distance as

$$\dot{z} = \dot{r}_{DM} \phi + r_{DM} (\dot{\lambda}_{TM} - \dot{\lambda}_{DM}) \tag{9}$$

$$\begin{aligned} \ddot{z} = & 2V_{DM} \frac{t_{fTM} - t_{fDM}}{t_{gDM}} \dot{\lambda}_{TM} + \cos(\gamma_D - \lambda_{DM}) u_D \\ & - \frac{r_{DM}}{r_{TM}} \cos(\gamma_T - \lambda_{TM}) u_T \\ & + \left[\frac{r_{DM}}{r_{TM}} \cos(\gamma_M - \lambda_{TM}) - \cos(\gamma_M - \lambda_{DM}) \right] u_M \end{aligned} \tag{10}$$

Choose $\mathbf{x} = [z \quad \dot{z} \quad \dot{\lambda}_{TM} \quad \dot{\lambda}_{DM}]^T$ as the state vector of the system. Then, the linearized kinematics can be written in a compact matrix form as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_D u_D + \mathbf{B}_T u_T + \mathbf{B}_M u_M \tag{11}$$

with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2V_{DM} \frac{\Delta t_f}{t_{gDM}} & 0 \\ 0 & 0 & \frac{2}{t_{gTM}} & 0 \\ 0 & 0 & 0 & \frac{2}{t_{gDM}} \end{bmatrix} \tag{12}$$

$$\mathbf{B}_D = \begin{bmatrix} 0 & \cos(\gamma_D - \lambda_{DM}) & 0 & -\frac{\cos(\gamma_D - \lambda_{DM})}{r_{DM}} \end{bmatrix}^T \tag{13}$$

$$\mathbf{B}_T = [0 \quad t_2 \quad t_3 \quad 0]^T \tag{14}$$

$$\mathbf{B}_M = \begin{bmatrix} 0 & m_2 & m_3 & \frac{\cos(\gamma_M - \lambda_{DM})}{r_{DM}} \end{bmatrix}^T \tag{15}$$

where $V_{ij} = -\dot{r}_{ij}$, $\Delta t_f = t_{fTM} - t_{fDM}$, and

$$\begin{aligned} t_2 &= -\frac{r_{DM}}{r_{TM}} \cos(\gamma_T - \lambda_{TM}) \\ t_3 &= -\frac{\cos(\gamma_T - \lambda_{TM})}{r_{TM}} \\ m_2 &= \frac{r_{DM}}{r_{TM}} \cos(\gamma_M - \lambda_{TM}) - \cos(\gamma_M - \lambda_{DM}) \\ m_3 &= \frac{\cos(\gamma_M - \lambda_{TM})}{r_{TM}} \end{aligned} \tag{16}$$

According to the concept of three-point guidance, the defender should stay on the LOS connecting the target aircraft and the attacking missile; hence, it can successfully capture the attacking missile before the target is intercepted by the attacker [21]. For this reason, the main purpose of this paper is to design an optimal cooperative guidance law that zeros the error distance while minimizing a performance index.

3. Optimal Problem Formulation

3.1. Implicit Cooperative Defense Problem

Since the objective of this paper is to protect the target from the attacking missile by zeroing z , the terminal constraint is

$$z(t_{fDM}) = 0 \tag{17}$$

Notice that the quadratic control energy consumption is directly related to the induced drag; hence, minimizing the quadratic energy is helpful to improving the terminal speed of the vehicle, enabling the improvement of the kill probability. Hence, we consider a quadratic energy performance index as

$$J = \frac{1}{2} \int_t^{t_{fDM}} u_D^2 d\tau \tag{18}$$

Consider u_T and u_M as two external known inputs to the defender. Then, the implicit cooperative defense problem for the defender is formulated as

$$\begin{aligned} \min_{u_D} J &= \frac{1}{2} \int_t^{t_{fDM}} u_D^2 d\tau \\ \text{s.t.} & \\ \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}_D u_D + \mathbf{G}_D \boldsymbol{\omega}, \quad z(t_{fDM}) = 0 \end{aligned} \tag{19}$$

where

$$\mathbf{G}_D = [\mathbf{B}_T \quad \mathbf{B}_M], \quad \boldsymbol{\omega} = [u_T \quad u_M]^T \tag{20}$$

Remark 1. Notice that the implicit cooperation defense problem is formulated to derive the guidance command of the defender with the known target maneuver u_T . Since error distance z is determined by the movement of three vehicles, the guidance strategy of the defense missile is related to the target aircraft. Therefore, this strategy can be regarded as a weak cooperation or an implicit cooperation.

3.2. Explicit Cooperative Defense Problem

The explicit cooperation between the target and the defender minimizes an arbitrary weighted quadratic energy consumption, i.e.,

$$\min_{u_D, u_T} J = \frac{1}{2} \int_t^{t_{fDM}} [R_D(\tau)u_D(\tau)^2 + R_T(\tau)u_T(\tau)^2] d\tau \tag{21}$$

where $R_D(t)$ and $R_T(t)$ are arbitrary positive weighting functions and are utilized to shape the guidance commands.

Consider u_M as an external known input to the defender. Then, the explicit cooperative defense problem for the defender and the target is formulated as

$$\begin{aligned} \min_{u_D, u_T} J &= \frac{1}{2} \int_t^{t_{fDM}} \mathbf{u}^T \mathbf{R} \mathbf{u} d\tau \\ \text{s.t.} & \\ \dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} + \mathbf{G} \omega, \quad z(t_{fDM}) = 0 \end{aligned} \tag{22}$$

where

$$\begin{aligned} \mathbf{B} &= [\mathbf{B}_D \quad \mathbf{B}_T], \quad \mathbf{u} = [u_D \quad u_T]^T \\ \mathbf{G} &= \mathbf{B}_M, \quad \omega = u_M \\ \mathbf{R} &= \begin{bmatrix} R_D & 0 \\ 0 & R_T \end{bmatrix} \end{aligned} \tag{23}$$

Remark 2. Since $R_D(t) \neq 0$ and $R_T(t) \neq 0$, the target and the defender perform explicit cooperation to defend the attacking missile. In this case, the target performs an inducing maneuver to cooperate with the defender. The benefit of explicit cooperation lies in the ability to shape the guidance commands to reduce the maneuverability demand of the defender. For example, if the defender has enough maneuvering capability, we can choose a smaller R_D to fully exploit the maneuverability of the defender; otherwise, a larger R_D would be a wise option.

4. Guidance Law Derivation

In this section, we derive the optimal cooperative defense guidance laws for both implicit and explicit cooperative scenarios.

4.1. Implicit Cooperative Defense Guidance Law

To solve the optimization problem formulated in Section 3.1, zero-effort transformation is first applied to reduce the system order. We denote Z as the zero-effort error distance, i.e., the terminal error distance when the defender performs no maneuvers from the current time onwards. With this in mind, the zero-effort error distance can be readily obtained as

$$Z = \mathbf{D} \Phi(t_{fDM}, t) \mathbf{x} + \int_t^{t_{fDM}} \mathbf{D} \Phi(t_{fDM}, t) \mathbf{G}_D \omega d\tau \tag{24}$$

where $\Phi(t_{fDM}, t)$ is the transition matrix associated with system (11), and

$$\mathbf{D} \triangleq [1 \quad 0 \quad 0 \quad 0]$$

Taking the time derivative of Equation (24) gives the dynamics of the zero-effort error distance as

$$\dot{Z} = b_D u_D \tag{25}$$

where

$$b_D = \mathbf{D} \Phi(t_{fDM}, t) \mathbf{B}_D \tag{26}$$

With this state transformation, the terminal constraint (17) can be replaced by

$$Z(t_{fDM}) = 0 \tag{27}$$

Then, the optimization problem (19) becomes finding u_D that minimizes performance index (18) subject to dynamics (25) and terminal constraint (27). For this optimization problem, the Hamiltonian function can be formulated as

$$H = \frac{1}{2}u_D^2 + \lambda \dot{Z} \tag{28}$$

where λ is the Lagrange multiplier.

Taking the time derivative of Equation (28) and applying the first-order optimality condition, i.e., $\partial H/\partial u_D = 0$, the optimal guidance command satisfies

$$u_D = -\lambda R_D^{-1} b_D \tag{29}$$

Substituting the preceding equation into Equation (25) and integrating it yields

$$Z(t_{fAD}) - Z(t) = -\lambda \int_t^{t_{fDM}} b_D^2(\tau) d\tau \tag{30}$$

Imposing constraint (27) in Equation (30), the Lagrange multiplier λ can be uniquely solved as

$$\lambda = \frac{Z(t)}{\int_t^{t_{fDM}} b_D^2(\tau) d\tau} \tag{31}$$

Substituting Equation (31) into Equation (29) gives the analytic guidance command as

$$u_D = -\frac{b_D}{\int_t^{t_{fDM}} b_D(\tau)^2 d\tau} Z(t) \tag{32}$$

For practical implementation, it is beneficial to formulate the guidance command in a more explicit form. From the definition of the system transition matrix, we can easily verify that

$$\mathbf{D}\Phi(t_{fDM}, t) = [1 \quad t_{gDM} \quad \varphi \quad 0] \tag{33}$$

where

$$\varphi = \frac{V_{DM}\Delta t_f}{2} \left[(t + \Delta t_f) \left(e^{\frac{2t_{gDM}}{t + \Delta t_f}} - 1 \right) - 2t_{gDM} \right] \tag{34}$$

Substituting Equation (33) into Equation (26) gives

$$b_D = \cos(\gamma_D - \lambda_{DM})t_{gDM} \tag{35}$$

Using Equation (33), the zero-effort error distance can be rewritten as

$$Z = z + \dot{z}t_{gDM} + \varphi\dot{\lambda}_{TM} + \int_t^{t_{fDM}} [b_T(\tau)u_T + b_M(\tau)u_M] d\tau \tag{36}$$

where

$$\begin{aligned}
 b_T &= \mathbf{D}\Phi(t_{fDM}, t) \mathbf{B}_T \\
 &= t_2 t_{gDM} + t_3 \varphi \\
 &= -\frac{r_{DM}}{r_{TM}} \cos(\gamma_T - \lambda_{TM}) t_{gDM} - \frac{\cos(\gamma_T - \lambda_{TM})}{r_{TM}} \varphi \\
 b_M &= \mathbf{D}\Phi(t_{fDM}, t) \mathbf{B}_M \\
 &= m_2 t_{gDM} + m_3 \varphi \\
 &= \left[\frac{r_{DM}}{r_{TM}} \cos(\gamma_M - \lambda_{TM}) - \cos(\gamma_M - \lambda_{DM}) \right] t_{gDM} \\
 &\quad + \frac{\cos(\gamma_M - \lambda_{TM})}{r_{TM}} \varphi
 \end{aligned} \tag{37}$$

Remark 3. From Equation (36), it is clear that calculating the zero-effort error distance requires the knowledge of target maneuver u_T and attacker maneuver u_M . Since the target aircraft can send information to the defender using a data link, the defender is capable of having access to the information of u_T . However, the future maneuver of the attacking missile is naturally unavailable to the defender, unless we can accurately identify the guidance law implemented by the attacker. For this reason, we use the current value of u_M in calculating Z and update it at every time instant. Notice that the proposed guidance law works in a closed-loop fashion and, hence, is helpful to compensate for the mismatch error.

4.2. Explicit Cooperative Defense Guidance Law

In this subsection, the guidance commands of both the target aircraft and the defender missile are simultaneously optimized. For the explicit cooperative guidance problem, the zero-effort error distance can be similarly derived as

$$Z = \mathbf{D}\Phi(t_{fDM}, t) \mathbf{x} + \int_t^{t_{fDM}} \mathbf{D}\Phi(t_{fDM}, \tau) \mathbf{G} \omega d\tau \tag{38}$$

which subsequently gives the dynamics of the zero-effort error distance as

$$\dot{Z} = \mathbf{b} \mathbf{u} \tag{39}$$

where

$$\mathbf{b} = [b_D \quad b_T] \tag{40}$$

With this state transformation, the explicit cooperative defense guidance optimization problem formulated in Section 3. B becomes

$$\begin{aligned}
 \min_{u_D, u_T} J &= \frac{1}{2} \int_t^{t_{fDM}} \mathbf{u}^T \mathbf{R} \mathbf{u} d\tau \\
 \text{s.t.} & \\
 \dot{Z} &= \mathbf{b} \mathbf{u}, \quad Z(t_{fAD}) = 0
 \end{aligned} \tag{41}$$

Consider the Hamiltonian function

$$H = \frac{1}{2} \mathbf{u}^T \mathbf{R} \mathbf{u} + \lambda \dot{Z} \tag{42}$$

where λ denotes the Lagrange multiplier.

From the first-order optimality condition, i.e., $\partial H / \partial \mathbf{u} = 0$, the optimal guidance command can be determined as

$$\mathbf{u} = -\lambda \mathbf{R}^{-1} \mathbf{b}^T \tag{43}$$

Substituting the preceding equation into Equation (41) and integrating it yields

$$Z(t) = \lambda \int_t^{t_{fDM}} \mathbf{b} \mathbf{R}^{-1} \mathbf{b}^T d\tau \tag{44}$$

Then, the Lagrange multiplier λ can be readily solved as

$$\lambda = \frac{Z(t)}{\int_t^{t_{fDM}} [R_D^{-1} b_D^2(\tau) + R_T^{-1} b_T^2(\tau)] d\tau} \tag{45}$$

On substitution of Equation (45) into Equation (43), the analytic explicit cooperative guidance law can be obtained as

$$\begin{aligned} u_D &= -\frac{R_D^{-1} b_D}{\int_t^{t_{fDM}} [R_D^{-1} b_D^2(\tau) + R_T^{-1} b_T^2(\tau)] d\tau} Z(t) \\ u_T &= -\frac{R_T^{-1} b_T}{\int_t^{t_{fDM}} [R_D^{-1} b_D^2(\tau) + R_T^{-1} b_T^2(\tau)] d\tau} Z(t) \end{aligned} \tag{46}$$

where b_D , b_T , and b_M are the same as those of the implicit cooperative scenario. The explicit form of the zero-effort error distance is

$$Z(t) = z + \dot{z}t_{gDM} + \phi \dot{\lambda}_{TM} + \int_t^{t_{fDM}} b_M(\tau) u_M d\tau \tag{47}$$

Remark 4. Similar to the implicit cooperation case, u_M is assumed as constant at every time instant in calculating Z . The benefit of the explicit cooperation strategy is that we can shape the guidance command of the target–defender team by choosing proper weighting functions R_D and R_T . We also demonstrate in numerical simulations that the explicit cooperation strategy helps to reduce energy consumption, compared with the implicit cooperative guidance law.

5. Simulations and Discussions

In this section, the performance of the proposed implicit and explicit cooperative guidance laws is evaluated through numerical simulations. We first numerically analyzed the characteristics of the proposed guidance algorithm. Then, existing cooperative CLOS and PNG were performed in the simulations for the purpose of performance comparison. The required initial conditions are summarized in Table 1. Notice that the guidance law of the attacking missile was chosen as PNG with the navigation gain being 3. The maximum accelerations of target and missiles were respectively limited as 30 m/s² and 300 m/s² in the simulations. In addition, since the simulations were performed under a constant speed model and the missile was treated as an ideal mass point, the normal wind was negligible.

Table 1. Initial Conditions.

Parameters	Values
$(x_{M_0} \ y_{M_0})$	(15 km 0 km)
$(x_{D_0} \ y_{D_0})$	(0 km 6 km)
$(x_{T_0} \ y_{T_0})$	(0 km 6 km)
V_M	600 m/s
V_D	600 m/s
V_T	300 m/s
γ_{M_0}	135°
γ_{D_0}	0°
γ_{T_0}	0°

5.1. Characteristics Analysis

In this subsection, the implicit cooperative defense guidance (ICDG) and explicit cooperative defense guidance (ECDG) were firstly simulated and compared with each other. The energy penalty weightings were chosen as $R_D = 1$ and $R_T = 1$, i.e., energy minimization was considered in the simulations. The simulation results, including vehicle trajectory, guidance command, and zero-effort error distance, obtained from ICDG and ECDG for the considered scenario are presented in Figure 2.

From Figure 2a, it is clear that the defender guided by the proposed guidance laws was capable of intercepting the attacking missile. This can also be verified by Figure 2b where the zero-effort error distance gradually converged to zero. The guidance commands in Figure 2c revealed that the required defender maneuverability of ECDG was smaller than that of ICDG. This can be attributed to the fact that the ECDG enabled the target aircraft to perform an inductive maneuver to lure the attacker, as can be seen from the target acceleration in Figure 2c. Hence, the ECDG algorithm provided more operational margins for the defender to cope with other disturbances and reduce the terminal miss distance. It is worth mentioning that the control energy consumption required by the defender of ICDG and ECDG were 7.60×10^4 and 7.05×10^3 , respectively. This again confirmed that the ECDG helped to reduce energy consumption and, thus, can improve the kill probability with higher speed.

We analyzed the effect of the energy penalty weighting on the guidance performance. Figure 3 presents the simulation results of ECDG with $R_T = 1$ and various $R_D = 0.5, 1, 2$. Notice that increasing R_D generated similar results to decreasing R_T . Hence, we kept R_T fixed in the numerical study. The quantitative comparison results of the required control effort of the defender and the target are shown in Table 2. The results indicated that increasing R_D helped to reduce the energy consumption and the required maneuver capability of the defender. Therefore, if the defender had enough maneuvering capability, we could choose a smaller R_D to fully exploit the maneuverability of the defender; otherwise, a larger R_D would be a wise option. However, it is worth pointing out that R_D cannot be arbitrarily large, since the maneuver of the target aircraft is physically limited.

Table 2. Control Effort of ECDG with Different R_D .

	Defender	Target
$R_D = 0.5$	$7.27 \times 10^3 \text{ m}^2/\text{s}^3$	$5.95 \times 10^2 \text{ m}^2/\text{s}^3$
$R_D = 1$	$7.05 \times 10^3 \text{ m}^2/\text{s}^3$	$9.62 \times 10^2 \text{ m}^2/\text{s}^3$
$R_D = 2$	$6.12 \times 10^3 \text{ m}^2/\text{s}^3$	$2.18 \times 10^3 \text{ m}^2/\text{s}^3$

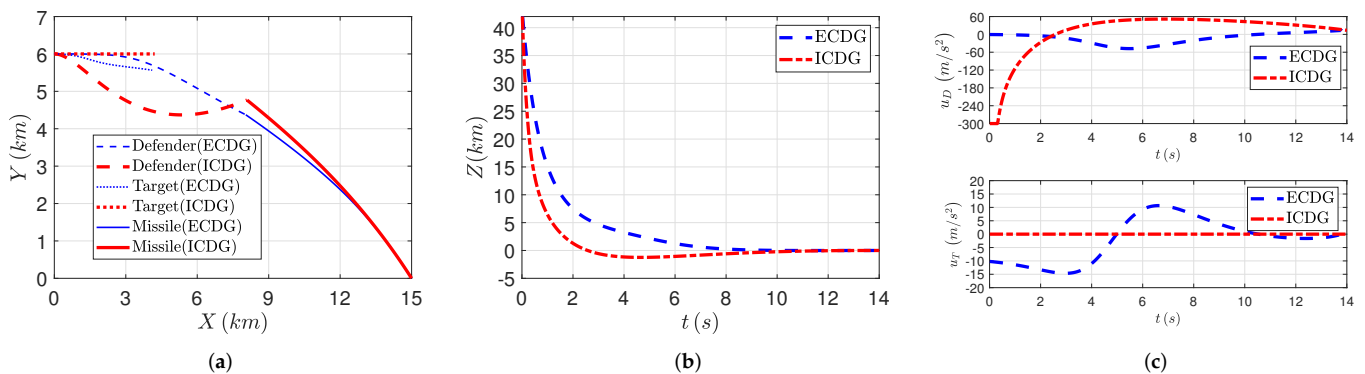


Figure 2. The simulation results of the proposed guidance law. (a) Trajectory, (b) Zero-effort error distance, (c) Guidance command.

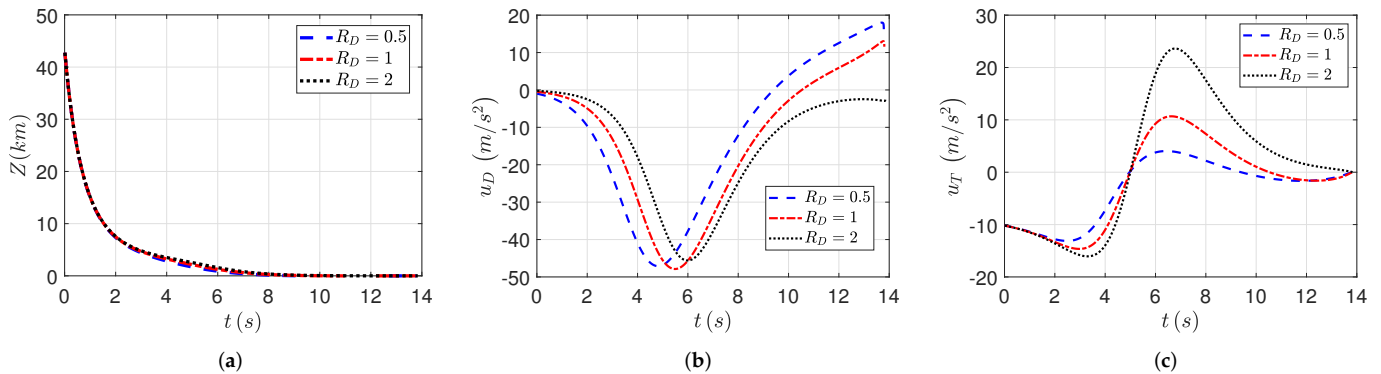


Figure 3. The simulation results of the proposed guidance law with different R_D . (a) Zero-effort error distance, (b) Guidance command of defender, (c) Guidance command of target.

5.2. Comparison with Existing Solutions

In this subsection, the ECDG with $R_T = R_D = 1$ was compared with the existing cooperative CLOS [24] and PNG to demonstrate the superiority of the proposed guidance law.

Figure 4 presents the comparison results obtained from different guidance laws. The results indicated that these three different guidance laws could successfully guide the defender to intercept the incoming missile and the error distance gradually converged to zero. However, we can observe from Figure 4c that the guidance command of CLOS was saturated at the beginning. This is because the cooperative CLOS is derived from an ideal geometry condition in which the first and second derivatives of the LOS angle among the three vehicles are equal. However, this assumption is invalid at the initial period, since $Z \neq 0$. Although the CLOS leverages a lead-lag compensator to regulate the angle error, tuning the design parameters depends on the application scenarios. The recorded control effort and maximum defender-to-attacker maneuverability ratio are summarized in Table 3. The results indicated that the proposed ECDG consumed less energy than the other two guidance laws. More importantly, the ECDG ensured successful interception with the smallest maneuverability demand. In conclusion, the proposed ECDG is more practical for aircraft protection applications, since the defender and the attacker usually have similar maneuver capability.

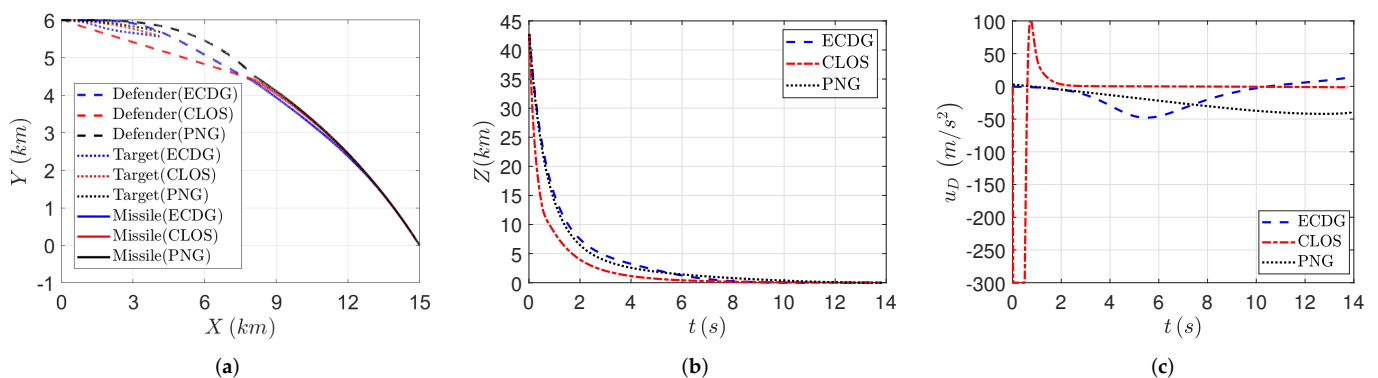


Figure 4. Comparison results of the proposed guidance law with PNG and CLOS. (a) Trajectory, (b) Zero-effort error distance, (c) Guidance command of defender.

Table 3. Quantitative Comparison Results.

	Control Effort	$ u_D/u_M _{max}$
ECDG	$7.05 \times 10^3 \text{ m}^2/\text{s}^3$	2.09
CLOS	$4.88 \times 10^4 \text{ m}^2/\text{s}^3$	15.57
PNG	$1.09 \times 10^4 \text{ m}^2/\text{s}^3$	3.06

6. Conclusions

An optimal guidance law for a target aircraft and its defender cooperatively intercepting an incoming missile was proposed based on three-body kinematics. By formulating a finite-time optimal control problem, the optimal defense guidance laws based on LOS triangle geometry were derived in both implicit and explicit cooperative forms. The obtained guidance command consisted of time-varying navigation gains and zero-effort error distance. The main feature of the proposed approach was to realize the cooperation in different levels. Numerical simulations with some comparisons clearly demonstrated the superiority of the proposed algorithm.

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