Using Instructional Materials to Overcome Students' Difficulties in Solving Problems Related to Pyramids

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

This paper remedies the inability of visual Art 2 students of Komfo Anokye Senior High School, Kumasi in calculating the perpendicular height correctly in a given pyramid and further use it to find the volume of the pyramid. Designed instructional materials were used to review the basic concepts like Pythagoras theorem before using activity method in teaching the actual topic- volume of a pyramid.
A pre-test was conducted to identify the specific areas for development and a post-test to assess the effectiveness of the intervention scheme. A sample of fifty students was selected from visual Art 2 of the school. The study concluded with suggestions and recommendations for the effective use of appropriate teaching and learning materials for positive results in teaching of mensuration especially, pyramids.

Keywords: Pythagoras theorem; pyramid; volume.

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1. INTRODUCTION

Mensuration as an aspect of mathematics is the scaling of the physical dimensions and volume of objects. It involves the science of measuring, that is, the science that studies how to represent geometrical quantities numerically, especially as to length, area, and volume. Adu (2004) also added that, mensuration involves finding lengths, areas and volumes of three-dimensional figures such as cylinder, pyramids, cone, prism etc.

Smith and Minton [1] on the topic, "Three dimensional figures" stated clearly that pyramid is a solid with many faces that has only one base which is a polygon. The name of a pyramid depends upon the shape of its base. For example a square pyramid has a square base, while a hexagonal pyramid has a six-sided base. The concept of pyramids has been applied in many fields across the globe. To mention few, a lot of monuments of the Egyptian Pharaohs and buildings have been built with ideas of pyramids. In the same line, most of churches within our environment have bell-towers, often topped with pyramid which has eight-sided base polygon. Also, in the Northern part of Ghana, most of the buildings and structures have been built in the shape of pyramids.

In the senior high school Core Mathematics syllabus, students are required in the second year to study pyramids, a sub-topic under mensuration. They are expected to be competent in solving problems in these areas: The height, Total surface area and the volume of pyramids, as well as finding the angles of the various angles associated with the pyramid. But these concepts most of time suffer complaints from students. Most of these complaints were against the abstract nature of teaching mathematics, which makes the subject challenging.

However, Hunt (1989) stressed that: "teaching all aspects of solid figures need concrete materials and then breaking down the topic into smaller units for easy understanding". This is also totally missing in our classrooms since the notion about the topic itself is formulae memorization, which can create a platform for many candidates to ignore question(s) under the topic by way of considering others which do not involve many rules.

The research work is aimed at introducing visual arts 2 students of Komfo Anokye senior high school in Kumasi of Ashanti region to a comprehensive and effective approach of solving problems related to pyramids. Thus, differentiate between the slant height and the true height as well as finding the volume of the pyramid.

McCarthy [2] emphasized the importance of the use of diagrams in the teaching of mathematics in all levels especially in the lower levels of formal education. He further says that teaching a particular topic under mathematics should be supported with diagrams for the students to understand the concept very well. This is in line with Chinese proverb saying: "when I hear, I forget, when I see, I remember and when I do I understand".

Rachlin (1992) also placed greater emphasis on helping children to think for themselves, learn through their own activities and to enjoy what they are doing. The strategy is driven partly by the problem themselves and partly by the pedagogy. They added that the problem context encourages students to explore and investigate the uniqueness of answers and the variety of solution paths.

To further buttress the above point, Yelan, as cited by Nuamah [3] was of the same view that there are many ways instructional materials can be used during instruction. However, each material has its own purpose and should be selected accordingly. By so doing students are likely to understand explanations, discussions and activities more quickly and recall them more completely and accurately. By working with well-structured Teaching-Learning Materials during practice, students will be able to recall important information better when they work independently in the calculations of a problem.

Ali Ayarebillah [4], researched into the problems of students' difficulty in answering questions on Mensuration specifically Pyramid at St. John Bosco Teacher Training College and found that, "more than 90% (of the final year students) did not attempt Mensuration questions in the end of year mock examinations, conducted internally by the college. However, after taking students through understanding relational concepts of mensuration using practical activity method, the situation improved drastically as 60% of the students were able to solve questions on pyramid without difficulty. He also emphasized that teaching and learning must be mutually participatory, as the use of teaching and learning materials will facilitate the concept of similarities

and differences among solids to enhance relational concept building.

Nuamah [3] also suggested in his work, the effect of the use of Teaching Learning Materials on the teaching and learning of solid Geometry at Swedru Secondary School. He found that, students had difficulties in 3-dimensional figures. But after using Teaching-Learning Materials in teaching the concept of solid Geometry, students understanding were enhanced and route learning was eliminated. He concluded that the data results obtained during Pre-test and Post-test shown improvement of the students' performance.

Abotsi [5], also found that, the mean mark for pre-lesson assignment and post–lesson assignment shown improvement of students understanding of concept of pyramids after using different approach of teaching of pyramids in his work "Alternative way of teaching pyramids ".

According to Mbowura [6], in his research, "using alternative way to solve students' difficulties in pyramids at E. P. Teacher Training College "revealed that, most students appreciate the innovation brought into the teaching of pyramids which made them very happy and felt very confident with the approaches used in solving problems on pyramids after the intervention. This was ascertain when he observed that the z-score was higher than the critical value, which implied that the hypothesis that the pretest mean was greater than the post-test, mean he rejected the pre-test.

2. MATERIAL AND METHODS

2.1 Population-Sample

The target population was visual Arts 2 class of Komfo Anokye senior high school (KAHIS) in Kumasi. The sample consisted of all the students in that class, where the problem was identified. The class consisted of fifty students. The average age of the selected class is projected to sixteen years.

2.2 Instrumentation

The research instruments used were interview and achievement test. The achievement test consists of pre-test and post-test. The interview was unstructured, and conducted in two phases. That is, before the pre-test and after the implementation of the intervention. Each was

composed of five items including follow ups where necessary.

The first item demanded the respondent's view on using Pythagoras theorems to find the missing length of right angle triangle. The second item also sort the respondent's view about the parts of square pyramid. The third item wanted to find out the challenges student uncounted in calculating the height of a given pyramid while the fourth item sourced for reason why students may not like answering questions related to pyramid.

The pre-test was aimed at knowing the strengths and areas for development whiles the post-test was also used in making inferences and drawing conclusions. The content of both tests were;

Question one was intended to test students' knowledge on using Pythagoras' theorem to find a missing length of right-angled triangle.

Question two was also used to test students' ability to distinguish between the slant height and slant edge.

The question three was aimed at testing students' ability to calculate the height of the pyramid whereas the last question was used to assess students' ability to find the volume of pyramid correctly. Samples of the pre-test and the post-test are shown in appendixes A and B respectively.

2.2.1 Administration of the research instrument

The pre-test was administered to students before the intervention to find out students' strengths and areas for development. The post-test was also conducted after the intervention to make inferences and draw conclusions. The pre-test and post-test questions were typed. It was made up of four (4) questions and marked out of 20 marks. They were conducted under strict examination conditions but not meant for their continuous assessment to avoid cheating and loosing its value. The results of the two tests were adequately recorded, tabulated and analyzed.

The first phase of the interview was conducted before the intervention whiles the second phase was done after the implementation of the intervention. The students were interviewed in turns. The unstructured interview was used

because the respondents were known to have been involved in the particular experiences, ie the intervention strategies which were relevant to the researcher in determine the level of students understanding of the concept of pyramids after the intervention. The responds from both interviews were not scored. It was purposely to find out how students had got the intervention activities.

2.3 The Actual Intervention

After a careful study of students' problems relating to pyramids, the researcher took the target group through the following sequence of intervention lessons.

- Activity-based teaching methods to review Pythagoras' theorem.
- Students must differentiate between the slant height and the true height using activity–based teaching.

- Students must calculate the true height and use it to find the volume of pyramid through activity method.

Under this we describe how the students are helped to overcome the problem they faced in solving problems related to pyramids.

2.3.1 Activity one- review of Pythagoras' theorem

Students were guided to draw a right triangle and labeled the vertices as shown in Fig. 1. They identified the side facing the angle 90^0 as the hypotenuse, the length of "h". With the referenced angle Ө, the line AC as the opposite side and the line AB as the adjacent, to the referenced angle are of the lengths "b" and "a" respectively. They were guided to construct squares on the arms of the right triangle based on the length of each side of the given right triangle as shown in Fig. 2.
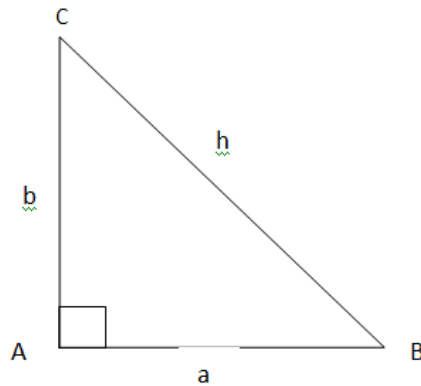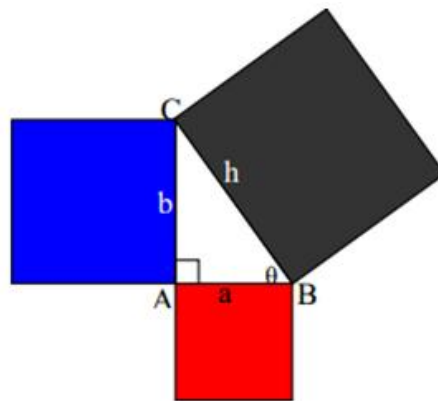
Fig. 1. Drawn of a right angle triangle

Fig. 2. Construction of squares on the arms of the right triangle

They were guided further to cut out each square from the arms of right triangle and then find the area of each cut out square.

Thus:

Area of red square + area of blue square = area of black square

Taking the length a, b and h as 3, 4 and 5 respectively into the above equation.

$$(a \times a) \quad + \quad (b \times b) \quad = \quad (h \times h)$$

$$a^2 \quad + \quad b^2 \quad = \quad h^2$$

Taking the length a, b and c as 3, 4 and 5 respectively into the above equation.

$$a^2 + b^2 = h^2$$
$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$
$$25 = 25$$

After a critical deduction from the above, the students finally generalized that in any right triangle, the square of the length of the adjacent side (a^2) added to the square of the length of the opposite side (b^2) equals the square of the length of the hypotenuse (h^2).

Symbolically,

$$|Adjacent|^2 + |Opposite|^2 = |Hypotenuse|^2$$
$$|AB|^2 + |AC|^2 = |BC|^2$$
$$a^2 + b^2 = h^2$$

2.3.2 Activity two

Getting students in groups of five (5) with 'improvised "dismantleable" pyramids', plain sheets, copper wire, rule, pencils, protractor and a worksheet, students follow the instructions on the worksheet. Firstly, students were guided to identify the various parts of "dismantleable pyramid." as shown below.

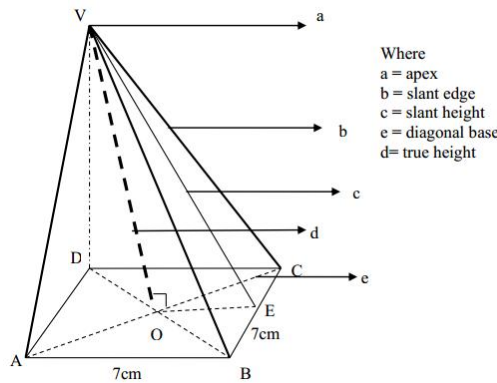

Where
a = apex
b = slant edge
c = slant height
e = diagonal base
d= true height

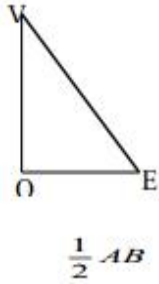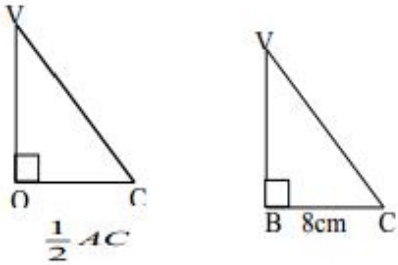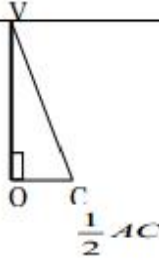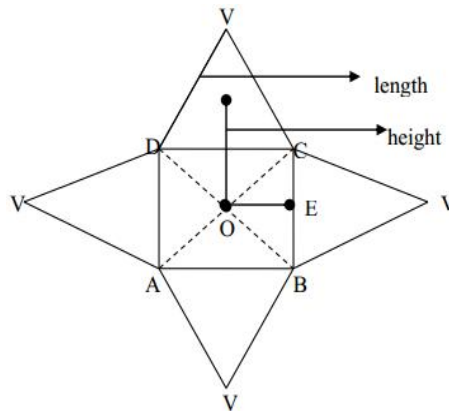**Fig. 3. Dismantleable square pyramid**

Fig. 4. Net of square pyramid

On the worksheet, students pull-out the rod pushed through the pyramid vertically representing the true height and record its length. They also measured and recorded the slant height and slant edge. They then compare the values and discover the differences.

The chart 1 below shows an example of students' work.

SIDE	DIAGRAM	LENGTH
SLANT HEIGHT	V, O, E $\frac{1}{2}AB$	VE=8.06cm
SLANT EDGE	V, O, C $\frac{1}{2}AC$ V, B 8cm C	VC = 9cm
TRUE HEIGHT	V, O, C $\frac{1}{2}AC$	OV= 7cm

Students identified and concluded that the slant height VE of a given pyramid is not equal to the slant edge VC and that of perpendicular height (True height) OV.

2.3.3 Activity three - The volume of a pyramid

In finding the volume of the pyramid, the students were taking through the below steps.

Step 1: The length of diagonal

Students were also asked to draw a right-angled triangle formed by two adjacent sides of the base and the diagonal of the "dismantleable pyramid". They used this to find the length of the diagonal, $\overline{AB}$ from ΔABD and use it to calculate the length OB, the base of ΔVOB.
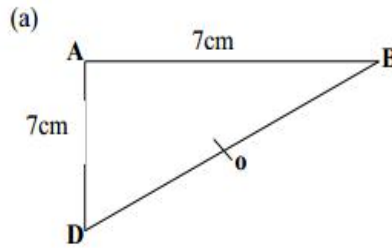
(a)

Fig. 5. Diagonal

The length of diagonal, B.D.

$$|AB|^2 + |AD|^2 = |BD|^2$$
$$7^2 + 7^2 = BD^2$$
$$49 + 49 = BD^2$$
$$98 = BD^2$$
$$\sqrt{98} = BD$$
$$\Rightarrow \qquad BD = 9.899$$

Step 2: The height of the Pyramid

Again they drew a right-angled triangle formed by the slant edge, the true height and half the diagonal of the base. This helps them to find the true height using the Pythagoras' theorem as shown below.
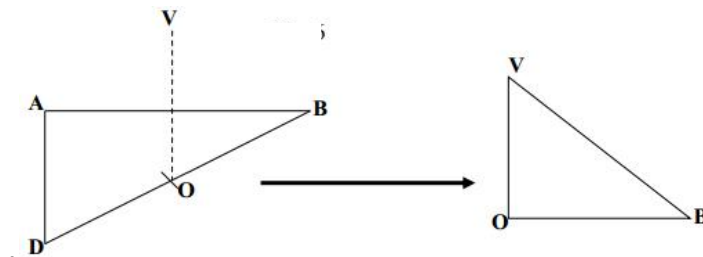

Fig. 6. Drawn a right-angled triangle formed by the slant edge

Therefore, $OB = 1/2(BD)$

$OB = 1/2(9.899)$

$OB = 4.9495$

(b) Finding the true height of the pyramid from Fig. 6 above

(Taking the slant edge to be 12 cm)

$$VB^2 = OB^2 + OV^2$$

Where $VB = 12$ and $OB=4.9495$

$$12^2 = (4.9495)^2 + OV^2$$
$$144 = 24.49755 + OV^2$$
$$OV^2 = 144 - 24.49755$$
$$OV^2 = 119.50245$$
$$OV = \sqrt{119.50245}$$
$$OV = 10.939317 \qquad\qquad OV = 10.93cm \text{ (2dp)}$$

Step 3: Calculating the volume of the pyramid:

Now, using the calculated height, OV =10.93cm, the students were guided to find the area of the base of the square pyramid.

Thus:

The Area of the base (B) = length × Breadth
$$= acm \times acm$$
$$= 7cm \times 7cm$$
$$= 49cm^2$$

The students were able to identify the volume of square pyramid as;

Volume of a pyramid is one third of the base area and the height.

The Angle between Face VBC and the Base

$$V = \frac{1}{3} \times B \times h$$

Where h = OV = 10.93cm and $B = 49cm^2$

$$V = \frac{1}{3}(49 \times 10.93)$$
$$= \frac{1}{3}(535.57) = 178.5233cm^3$$
$$= 178.52cm^3$$

Hence, the volume is $178.52cm^3$

Activity – four

Further, students were guided to use copper wire with L-shape to trace the angles between: the slant edge and the base, the slant face and the base, and any two opposite face and compare the three values.
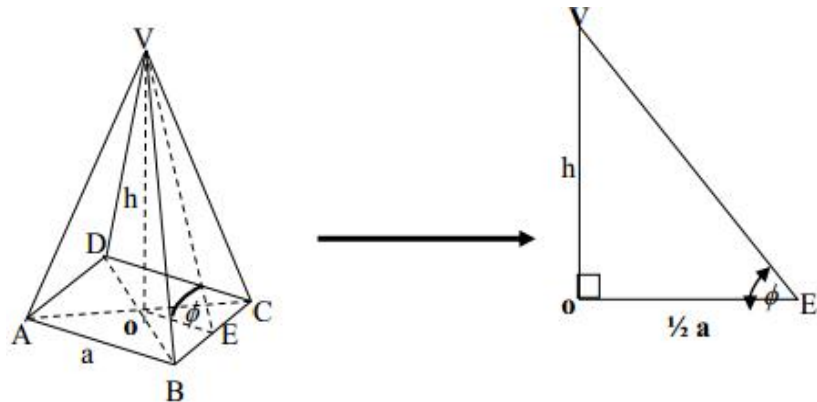

Fig. 7. Angle between Face VBC and the Base
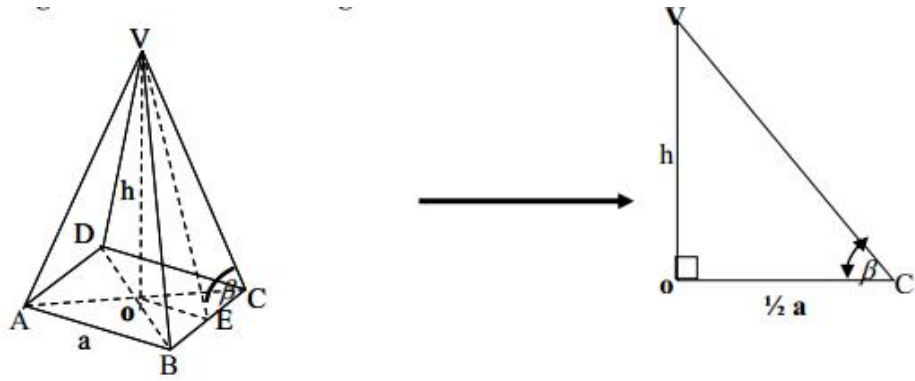
The Angle between the Slant Edge and the Base

Fig. 8. Angle between the Slant Edge and the Base

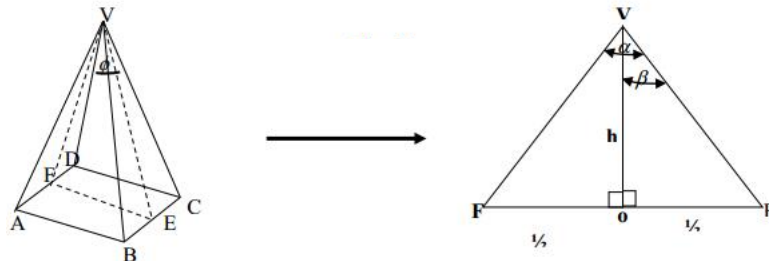**The Angles between the Faces VBC and VAD**

Fig. 9. Angles between the Faces VBC and VAD

3. DISCUSSION AND ANALYSIS OF RESULTS

3.1 Discussion of the Pre-Test and Post-Test

The Tables 1 and 2 show results of pre-test and post-test respectively conducted in the class of 50 students.

3.2 Discussion of the Data

Question 1: Question one was aimed at testing students' knowledge on using Pythagoras' theorem to find the length of the hypotenuse. In the pre-test, 41 out of 50 students had question one correct representing 82% whereas in the post-test, 48 out of 50 students answered question one correct representing 96%.

Question 2: Question two was also used to test students' ability to distinguish between the slant height and slant edge. For the pre-test, 21 students had question two correct representing 42% and for the post-test, 46 students had answers correct representing 92%.

Question 3: For question three, students were asked to find the height of the pyramid of which 13 out 50 had the question correct representing 26% in the pre-test, whereas in the post-test, 37 had it correct representing 74%.

Question 4: Question four was used to assess students' ability to find the volume of pyramid. In the pre-test 24 students had question four correct representing 48% whereas in the post-test, 38 students had the question correct representing 76%.

The Tables 3 and 4 show the summary of the marks obtained against the number of students who got that mark in both pre-test and post- test.

Table 1. Correct and wrong answers obtained by students in pre-test

Question No	No. of Students obtaining wrong answer (A)	Percentage of (A)	No. of Students obtaining correct answer (B)	Percentage of (B)
1	9	16	41	82
2	29	58	21	42
3	31	62	19	38
4	26	52	24	48

Table 2. Correct and wrong answers obtained by students in post-test

Question No	No. of Students obtaining wrong answer (A)	Percentage of (A)	No. of Students obtaining correct answer (B)	Percentage of (B)
1	2	4	48	96
2	4	8	46	92
3	13	26	37	74
4	12	24	38	76

Table 3. Summary of marks distribution of students in the pre-test

Marks	Frequency (f)	Midpoint (x)	Fx
0-2	9	1	9
3-5	11	4	44
6-8	18	7	126
9-11	6	10	60
12-14	4	13	52
15-17	2	16	32
18-20	0	19	0
	$\sum f = 50$		$\sum fx = 323$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} = \frac{323}{50} = 6.46$$

Table 4. Summary of marks distribution of students in the post-test

Marks	Frequency (f)	Midpoint (x)	Fx
0-2	1	1	1
3-5	5	4	20
6-8	6	7	42
9-11	9	10	90
12-14	7	13	91
15-17	14	16	221
18-20	8	19	152
	$\sum f = 50$		$\sum fx = 620$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} = \frac{620}{50} = 12.4$$

Below are pictorial representation of the pre-test and post-test results in the form of bar chart Fig. 10 and Fig. 11 respectively. The mean mark for the pre-test and post-test were 6.46 and 12.4 respectively.
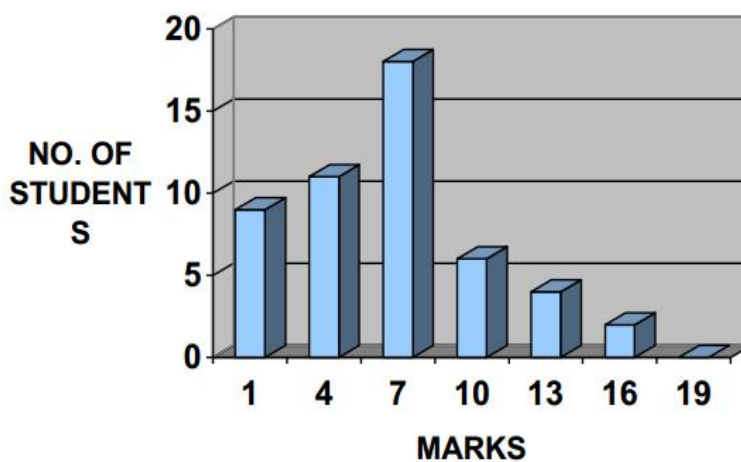

Fig. 10. Pre test results
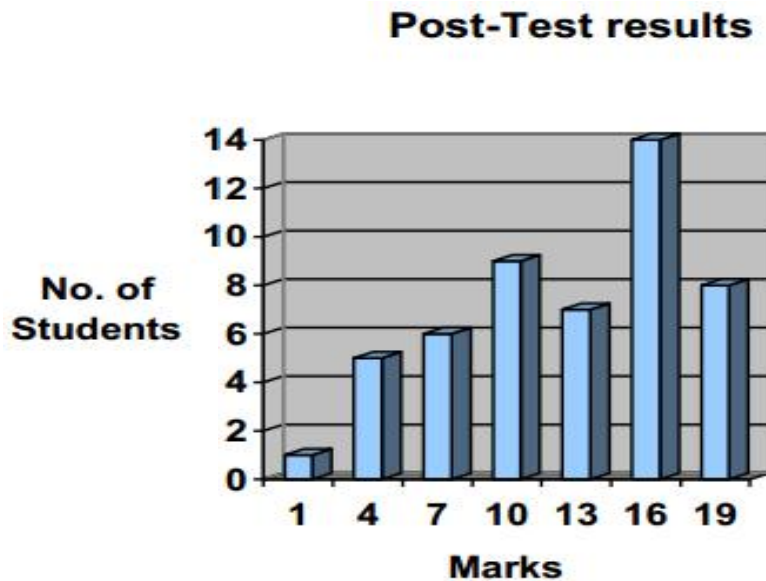
Post-Test results

Fig. 11. Post test results

3.3 Findings

During the pre-test, the performance above average was 24% and that of post-test was 76% of the total class of 50 students. The mean mark too wasn't far from the truth that students' performance has been improved after the intervention strategies because the mean mark for the pre-test was 6.46 and the post-test was also 12.4. The above results obtained indicated that the use of teaching and learning materials through activity-based teaching method has brought about substantial improvement in students' performance.

4. CONCLUSION

With reference to the results obtained from the pre-test and post-test stages, it could be concluded that students' performance have improved after the implementation of intervention. I am also of the view that the students poor performance in the pre-test was not due to their academic handicap or laziness but because the students were denied the necessary concepts needed to lead them to a better understanding. Moreover, the implementation of the intervention strategies brought a lot of influence on students' performance; however, they will perform better if they were adopted as early as practicable. It must be emphasized that students learn better when they are directly involve in the lesson,

therefore teaching and learning must be mutually participatory. The teacher must not formulate rules and formulae for students to "chew and pour" when demanded.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX A

Pre-test questions

1. A triangle FGH has a right-angle at G and the lengths are in centimeters $|GH| = 8cm$

And $|GF| = 12cm$. Find the length of $|FH|$
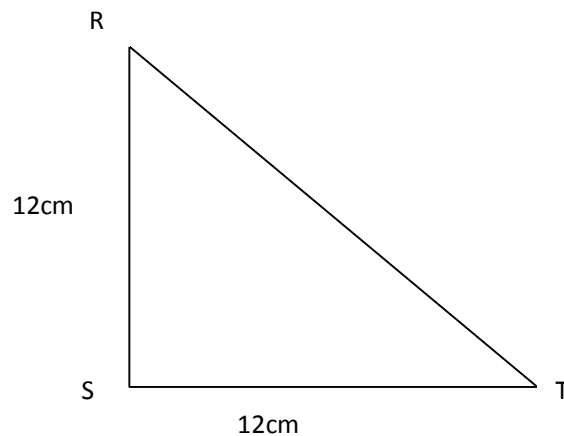
F

12cm

G   8cm   H
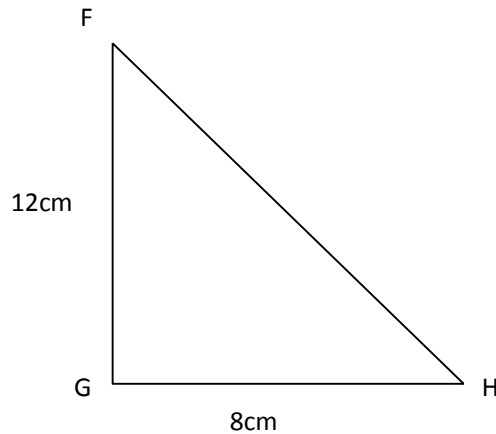
2. A pyramid $VEFGH$ has a square base of 6cm, if the volume of the pyramid is $60cm^3$. Find:

   i)   the height of the pyramid.
   ii) the slant height of the pyramid.
   iii) the length of the slant edge.

3. A pyramid with vertex O stands on a square base ABCD and |OA|= |OC|= |OD|=|AB|= 5cm. Calculate the height of the pyramid.
4. A pyramid VABCD has a square base of side 7cm and each side of the slant edge is 8cm. Calculate the volume of the pyramid to 2 significant figures.

APPENDIX B

Post-test questions

1. A triangle RST has a right-angle at R and the length of |RT| and |ST| are 12cm and 13cm respectively. Find the length of RS

R

12cm

S   12cm   T

2. A pyramid VABCD has a square base of 8cm and if the volume of the pyramid is $150cm^3$. Find:

   i) The height of the pyramid.
   ii) The slant height of the pyramid.
   iii) The length of the slant/slopping edge.

3. ABCD is a right square pyramid with vertex O, such that |OA|= |OB|= |OC|= |OD|= 30cm and |AB|=25cm. Calculate the height of the pyramid.
4. A pyramid VABCD has a square base of side 8cm and each side of the slant edge is 9cm. Calculate the volume of the pyramid to 2 significant figures.