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Development of Mass – Size Particle Reduction Operations Postulates Using Empirical – Analytical Approaches

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Aims: This paper attempts to utilize empirical and analytical approaches to develop equations and postulates for energy and power requirement in mass – size reduction operations.
Study Design: The study is based on a combination of analytical approach and empirically existing energy model obtained in a study of milling palm nut shells into fragments using static nut cracker.
Place and Duration of Study: The empirical model used in this study was developed in 2014 in the University of Benin, Nigeria and the analytical approach employed for the study to obtain mass – size particle reduction models and postulates were achieved in September, 2020 in the University of Uyo, Nigeria.
Methodology: In this task, the crushing efficiency, mechanical efficiency, energy and power requirements were considered based on fundamental principles coupled with the usage of the empirically developed minimum energy model for mass –size reduction operations.
Results: The Equations (59), (60), (61), (62) and (63) were developed. Equations (59), (60) and (62) are in the form of bond's energy equation but may differ in the value of the constant. This may be because the properties (density, thickness or diameter of the particle) of material and machine efficiency might easily be obtained and used to evaluate; and likely achieved an improved

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assessment of mass – size reduction of a given material.
Conclusion: Further analysis has led to the development of postulates presented in the conclusion part of this paper which may govern the mass – size reduction operations of particle.

Keywords: Energy; power; mass – size; equations; postulates.

1. INTRODUCTION

In particle size reduction, the basis is by considering the differential energy dE required to produce a small change in size dx of a unit of the material as:

$$\frac{dE}{dx} \propto x^{-n} \quad (1)$$

Some researchers such as Kicks, Rittingers' and Bonds have their energy equations on this basis for size reduction operation; where n = ½, 1, 2 for Bonds, Kicks and Rittingers energy equations respectively which could be expressed [1-7] as:

$$E_k = K_k \ln \frac{d_1}{d_2} \quad (2)$$

$$E_R = K_R \left[\frac{1}{d_2} - \frac{1}{d_1}\right] \quad (3)$$

$$E_b = K_b \left[\frac{1}{d_2^{1/2}} - \frac{1}{d_1^{1/2}}\right] \quad (4)$$

Where, E = energy required for size reduction, K_k, K_R and K_b are Kick's, Rittinger's and Bond's constants respectively.

d_1 = initial particle size
d_2 = average particle size after crushing (grinding)

$$K_b = 0.3162\, w_i \quad (5)$$

w_i = work index

The Kick's, Rittinger's and Bond's laws are suitable respectively for coarse, fine and intermediate particle obtained after grinding. However, the Kick's and Rittinger's has serious limitations in its usage as it is only applicable to limited particle ranges; and their constant could be obtained by carrying out experiments on the machine and material that is to be used. The Bond's work of 1952 is more realistic in approach as the working index is based on gross energy in kilowatt hour per kilogram (kWh/kg) of the material that is fed into the machine; such that 80% of the reduced size of the material from large size should be able to pass through 100 μm screen. The Bond's energy equation constant K_b depends on the type of machine and nature of the materials to be crushed [8]. In view of these challenges, Antia and co-researchers in 2014 when developing a machine for cracking of palm nuts observed that further fragmentation of shell particle sizes with little or no objectionable damage to kernels; in order to have easy recovery of kernels from shell particles when subjected to sieve aperture sizes was possible [9-11]. In this circumstance, the energy required for the shell fragmentation into smaller sizes was viewed in terms of mass than size. This is because the shape, thickness and nature of the palm nut shells vary. Hence, using the basis of mass with respect to its surface area was considered to likely give better result from the energy that would be modeled for size reduction operation. In this regard, the differential energy dE that is involved in producing a small change dA in area of a unit material could be expressed as a power function of area [12].

$$\frac{dE}{dA} \propto A^{-n} \quad (6)$$

In considering mass – size reduction operations, the minimum energy (E_{min}) required for the shell fragmentation was found to be proportional to the square root of the shell mass (M) and was expressed [6] as:

$$E_{min} = 2BM^{1/2} \quad (7)$$

Where, B is a constant with unit in $kg^{1/2}m^2s^{-2}$ and for palm nut shell B = 5.75$kg^{1/2}m^2s^{-2}$ while

E_{min} is in Joules

However, the energy evaluation in units of J/kg and kWh/kg may be considered necessary with reference to size reduction operations that would be based on Equation (7). This approach would possibly give a better assessment of the minimum energy requirement for mass – size reduction operations if compared with the existing energy equations for size reduction operations. The minimum power requirement may also be obtained based on the modeled energy equations. Therefore, in this study, postulates would be developed based on model equations obtained for energy and power required during mass – size reduction operations.

2. METHODOLOGY

In developing the minimum energy in J/kg and kWh/kg and minimum power in J/s or kW required for mass – size particle reduction operation; the following steps may be considered:

2.1 Energy Utilized

Let us denote the following energy during mass – size reduction operations.

i. The surface energy created per unit area of the material, J/m^2 = E_{sea}
ii. The energy absorbed per unit area of the material J/m^2 = E_{eaa}
iii. The surface energy per unit mass of the material J/kg = E^*_{sea}
iv. The energy input per unit area of the material, J/m^2 = E_{eia}
v. The mass of the product per unit area, kg/m^2 = M_{pa}
vi. The mass of the feed per unit area, kg/m^2 = M_{fa}

2.2 Crushing Efficiency

The crushing efficiency $_{cf}$ may be expressed as:

$$_{cf} = \frac{E_{sea}}{E_{eaa}} \tag{8}$$

But,

E_{sea}= E^*_{sea} × mass per unit area of the material, kg/m^2 (9)

During crushing, E_{sea} for mass – size reduction of feed to product may be written based on Equation (9) as:

$$E_{sea} = E^*_{sea}[M_{pa} - M_{fa}] \tag{10}$$

Equation (8) may be re-written as:

$$_{cf} = \frac{E^*_{sea}\,[M_{pa} - M_{fa}]}{E_{eaa}} \tag{11}$$

2.3 Mechanical Efficiency

The mechanical efficiency may be given as:

$$_{mf} = \frac{\text{output}}{\text{total input}} = \frac{E_{eaa}}{E_{eia}} \tag{12}$$

Substitute Equation (11) into Equation (12)

$$_{mf} = \frac{E^*_{sea}}{_{cf}\,E_{eia}}\,[M_{pa} - M_{fa}] \tag{13}$$

2.4 The Total Energy Input per Unit Area of the Material

The total energy input per unit area of the material E_{eia} may be expressed from Equation (13) as:

$$E_{eia} = \frac{E^*_{sea}}{_{cf}\ _{mf}}\,[M_{pa} - M_{fa}] \tag{14}$$

2.5 The Total Energy Input per Unit Mass of the Material

Based on Equation (14), the total energy input per unit mass of the material, J/kg may be obtained as:

$$E_{eia} \times S_A = \frac{E^*_{sea}}{_{cf}\ _{mf}}\,[M_{pa} - M_{fa}]\,S_A \tag{15}$$

Where, S_A = specific surface area, m^2/kg = $\frac{\text{surface area of particle}}{\text{mass of particle}}$

Let's denote: $E_{eia} \times S_A = E_{OA}$ (16)

Hence, the total energy input per unit mass, J/kg denoted as E_{OA} may be expressed as:

$$E_{OA} = \frac{E^*_{sea}}{_{cf}\ _{mf}}\,[M_{pa} - M_{fa}]\,S_A \tag{17}$$

2.6 The Units of E_{OA} in kWh/kg

The units of E_{OA} in kWh/kg may be expressed as:

i. $$E_{OA} = \frac{E^*_{sea}}{_{cf}\ _{mf}}\,[M_{pa} - M_{fa}]\,\frac{u^2t}{\bar{m}} \tag{18}$$

Where, $S_A = \frac{u^2t}{\bar{m}}$ (19)

u = velocity of the particle, m/s
t = time of mass – size reduction operation, s
$\bar{m}$ = mass flow rate of particle, kg/s

ii. The Equation (18) may alternatively be expressed in unit of kWh/kg as:

$$E_{OA} = \frac{E^*_{sea}}{_{cf}\ _{mf}}\,[M_{pa} - M_{fa}]\,\frac{M_p}{\rho_m Dt} \times \frac{1}{\bar{m}} \tag{20}$$

Where, $u^2t = \frac{M_p}{\rho_m Dt}$ (21)

M_p = mass of the material, kg

ρ_m = density of the material, kg/m^3
t = time required for the mass – size reduction operation, s
D = diameter of the material, m

2.7 The Energy E_{OA} in Terms of Mass – Size Reduction Operation

The energy E_{OA} in terms of mass – size reduction operation may further be evaluated with respect to particle size diameter of feed D_{fa} and product D_{pa} as follows:

M_{pa}, M_{fa} are mass per unit area of product and feed respectively

$$M_{pa} = \frac{\text{mass of particle,kg}}{\text{area of the particle,m}^2} \quad (22)$$

$$\text{mass} = \text{density } (\rho_m) \times \text{volume of the particle } (V_m)$$

If we assumed that the particles are spherical, then for sphere

$$\text{Volume} = {}^4/_3\,\pi r^3 \quad (23)$$

$$\text{Area} = 4\pi r^2 \quad (24)$$

Hence,

$$M_{pa} = \frac{\rho_m\,({}^4/_3\pi r_{pa}^3)}{4\pi r_{pa}^2} = \frac{\rho_m\, r_{pa}}{3} = \frac{\rho_m\, D_{pa}}{6} \quad (25)$$

Where, $D_{pa} = 2r_{pa}$ = diameter of particle, m
r_{pa} = radius of the particles, m

Similarly,

$$M_{fa} = \frac{\rho_m\, D_{fa}}{6} \quad (26)$$

Substitute Equations (25) and (26) into Equation (17)

$$E_{OA} = \frac{E^*_{sea}\rho_m}{cf\ mf}\left[\frac{D_{pa}}{6} - \frac{D_{fa}}{6}\right] S_A \quad (27)$$

$$E_{OA} = \frac{E^*_{sea}\rho_m}{6\ cf\ mf}\left[D_{pa} - D_{fa}\right] S_A \quad (28)$$

But, E^*_{sea} is energy per unit mass, J/kg; therefore considering Equation (7)

$$E^*_{sea} = \frac{E_{min}}{M_p} \quad (29)$$

Where, E_{min} is considered as the minimum energy for mass – size reduction operation, J

M_p = mass of the material, kg

$$E_{min} = 2BM_p^{1/2}$$

$$E^*_{sea} = \frac{2BM_p^{1/2}}{M_p} = 2BM_p^{-1/2} \quad (30)$$

$$M_p = \rho_m V_m \quad (31)$$

Where, V_m = volume of the particle, m^3

For a spherical particle,

$$V_m = {}^4/_3\,\pi\frac{D^3}{8} = \frac{\pi\, D^3}{6} \quad (32)$$

From Equation (30),

$$E^*_{sea} = 2B\left[\rho_m \pi \frac{\pi\, D^3}{6}\right]^{-1/2} \quad (33)$$

$$E^*_{sea} = 2B\left[\frac{\rho_m \pi}{6}\right]^{-1/2}[D^3]^{-1/2} = \frac{2B}{\left[\frac{\rho_m\pi}{6}\right]^{1/2} D^{3/2}} \quad (34)$$

$$E^*_{sea} = \frac{2B}{\rho_m^{\ 1/2}\left[\frac{\pi}{6}\right]^{1/2} D^{3/2}} \quad (35)$$

Substitute Equation (35) into Equation (28)

$$E_{OA} = \frac{2B\,\rho_m}{\rho_m^{\ 1/2}\left[\frac{\pi}{6}\right]^{1/2} D^{3/2}\,(6\ cf\ mf)}\left[D_{pa} - D_{fa}\right] S_A \quad (36)$$

$$= \frac{2B\,\rho_m^{\ 1/2}}{\left[\frac{\pi}{6}\times 36\right]^{1/2} D^{3/2}\ cf\ mf}\left[D_{pa} - D_{fa}\right] S_A \quad (37)$$

$$= \frac{2B\,\rho_m^{\ 1/2}}{[6\pi]^{1/2}\ cp\ mf}\left[\frac{D_{pa}}{D_{pa}^{3/2}} - \frac{D_{pa}}{D_{fa}^{3/2}}\right] S_A \quad (38)$$

$$= \frac{2B\,\rho_m^{\ 1/2}}{[6\pi]^{1/2}\ cp\ mf}\left[D_{pa}^{-1/2} - D_{fa}^{-1/2}\right] S_A \quad (39)$$

$$= \frac{2B\,\rho_m^{\ 1/2}}{[6\pi]^{1/2}\ cp\ mf}\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] S_A \quad (40)$$

Hence, (i) the units of E_{OA} in J/kg based on Equation (17) may be expressed as:

$$E_{OA} = 2B\,\rho_m^{\ 1/2}\left[\frac{1}{[6\pi]^{1/2}\ cf\ mf}\right][S_A]\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (41)$$

(ii) the unit of E_{OA} in kWh/kg based on Equation (18) could be expressed as:

$$E_{OA} = 2B\,\rho_m^{\ 1/2}\left[\frac{1}{[6\pi]^{1/2}\ cf\ mf}\right]\left[\frac{u^2 t}{\bar{m}}\right]\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (42)$$

(iii) the unit of E_{OA} in kWh/kg based on Equation (20) may be written as:

$$E_{OA} = 2B\,\rho_m{}^{1/2}\left[\frac{1}{[6\pi]^{1/2}\ cp\ mf}\right]\left[\frac{M_p}{\rho_m D t\,\bar{m}}\right]\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (43)$$

$$E_{OA} = 2B\,\rho_m{}^{-1/2}\left[\frac{1}{[6\pi]^{1/2}\ cp\ mf}\right]\left[\frac{M_p}{\bar{m}t}\right]\frac{1}{D}\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (44)$$

$$E_{OA} = 2B\,\rho_m{}^{-1/2}\left[\frac{1}{[6\pi]^{1/2}\ cp\ mf}\right]\left[\frac{M_p}{\bar{m}t}\right]\left[\frac{1}{D_{pa}^{3/2}} - \frac{1}{D_{fa}^{3/2}}\right] \quad (45)$$

But, $\frac{M_p}{t} = \bar{m}$

Hence, Equation (45) may also be expressed as:

$$E_{OA} = 2B\,\rho_m{}^{-1/2}\left[\frac{1}{[6\pi]^{1/2}\ cp\ mf}\right]\left[\frac{1}{D_{pa}^{3/2}} - \frac{1}{D_{fa}^{3/2}}\right] \quad (46)$$

2.8 The Power Required for Size Reduction Operation P_{cp}

The power required for size reduction operation P_{cp} in J/s may be expressed as:

$$P_{cp} = \frac{\text{total input energy per unit mass,J/kg}}{\text{mass flow rate,kg/s}} = \frac{\text{energy,J}}{\text{time,s}} \quad (47)$$

Now, in terms of E_{OA}, the power P_{cp} may be written as:

$$P_{cp} = E_{OA}\,(\bar{m}) \quad (48)$$

From Equation (42), the power P_{cp} in J/s or W or kW may be expressed based on Equation (48) as:

$$P_{cp} = 2B\,\rho_m{}^{1/2}\left[\frac{1}{[6\pi]^{1/2}\ cf\ mf}\right][u^2 t]\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (49)$$

From Equation (46), the power P_{cp} may be written based on Equation (48) as:

$$P_{cp} = 2B\,\rho_m{}^{-1/2}\left[\frac{1}{[6\pi]^{1/2}\ cf\ mf}\right][\bar{m}]\left[\frac{1}{D_{pa}^{3/2}} - \frac{1}{D_{fa}^{3/2}}\right] \quad (50)$$

3. RESULTS AND DISCUSSION

The results of the steps followed in developing new models for mass – size reduction operation revealed that:

(i) in J/kg based on Equation (41) may be written as:

$$E_{OA} = K_{OA_1}\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (51)$$

Where, $K_{OA_1} = \frac{2B\,\rho_m{}^{1/2}}{cf\ mf}\left[\frac{1}{[6\pi]^{1/2}}\right]S_A$

$$= \frac{2B\,\rho_m{}^{1/2}}{cf\ mf}[0.2304]S_A \quad (52)$$

(ii) in kWh/kg based on

(a) Equation (42) may be expressed as:

$$E_{OA} = K_{OA_2}\left[\frac{1}{D_{pa}^{1/2}} - \frac{1}{D_{fa}^{1/2}}\right] \quad (53)$$

Where, $K_{OA_2} = \frac{2B\,\rho_m{}^{1/2}}{cf\ mf}[0.2304]\frac{u^2 t}{\bar{m}}$ (54)

(b) Equation (46) may be given as:

$$E_{OA} = K_{OA_3}\left[\frac{1}{D_{pa}^{3/2}} - \frac{1}{D_{fa}^{3/2}}\right] \quad (55)$$

Where, $K_{OA_3} = \frac{2B\,\rho_m{}^{-1/2}}{cf\ mf}[0.2304]$ (56)

It is observed that Equations (51) and (53) resemble Bond's energy Equations (4) and (5)

However, if the crushing efficiency and mechanical efficiency are 97.15% and 75% respectively or 80% and 91.08% respectively the value of $\frac{0.2304}{cf\ mf}$ in constant K_{OA_1}, K_{OA_2} and K_{OA_3} becomes 0.3162 which is a value in the Bond's energy equation expressed in Equations (4) and (5).

The sphericity of particle diameter of the feed and product may be necessary to be applied to obtain a more realistic diameter of the particle.

Since the particles are considered to be spherical, then the D_{pa} and D_{fa} need to be expressed with respect to sphericity value (S_p) of the material as:

$$D_{vsp} = D_{pa}S_p \tag{57}$$

$$D_{vsf} = D_{fa}S_p \tag{58}$$

Based on Equations (57) and (58), Equation (51) for energy in J/kg and Equations (53) and (55) for energy in kWh/kg and Equations (49) and (50) for power in J/s or kW would be written as follows:

(i) For Equation (51):

$$E_{OA} = K_{OA_1}\left[\frac{1}{D_{vsp}^{1/2}} - \frac{1}{D_{vsf}^{1/2}}\right] \tag{59}$$

(ii) For Equation (53):

$$E_{OA} = K_{OA_2}\left[\frac{1}{D_{vsp}^{1/2}} - \frac{1}{D_{vsf}^{1/2}}\right] \tag{60}$$

(iii) For Equation (55):

$$E_{OA} = K_{OA_3}\left[\frac{1}{D_{vsp}^{3/2}} - \frac{1}{D_{vsf}^{3/2}}\right] \tag{61}$$

Where, E_{OA} may be referred to as Orua Antia's Energy Equation and K_{OA_1}, K_{OA_2} and K_{OA_3} as Orua Antia's Energy Equation Constants for mass – size particle reduction operations.

(iv) For Equation (49) based on Equations (57) and (58)

$$P_{cp} = K_{OA_2}\bar{m}\left[\frac{1}{D_{vsp}^{1/2}} - \frac{1}{D_{vsf}^{1/2}}\right] \tag{62}$$

(v) For Equation (50) based on Equations (57) and (58)

$$P_{cp} = K_{OA_3}\bar{m}\left[\frac{1}{D_{vsp}^{3/2}} - \frac{1}{D_{vsf}^{3/2}}\right] \tag{63}$$

4. CONCLUSION

Based on Equations (59), (60), (61), (62) and (63), it could be postulated that:

(1) The minimum energy in kilowatt hour per kilogram required for mass – size particle reduction operation is:

(a) Proportional to (i) the product of time and square of the particle velocity and (ii) reciprocal of particle mass flow rate and square root of the particle diameter

OR

(b) Proportional to the reciprocal of the cube of square root of the particle diameter

(2) The minimum energy in Joules per kilogram required for mass – size particle reduction operation is proportional to particle specific surface area and the reciprocal of the square root of the particle diameter.

(3) The minimum power in Joules per second required for mass – size particle reduction operation is:

(a) Proportional to the product of particle mass flow rate and reciprocal of the cube of the square root of the particle diameter

OR

(b) Proportional to (i) reciprocal of the square root of the particle diameter and (ii) the product of the time and square of the particle velocity

Moreover, the value of the Orua Antia Energy Equation Constants K_{OA_1}, K_{OA_2} and K_{OA_3} primarily depend on the efficiency of the machine, density and thickness of the material.

COMPETING INTERESTS

Author has declared that no competing interest exists.

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