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Study of Compactons and Solitons Using Finite Element Method

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Authors' contributions

This work was carried out in collaboration between all authors. Author SAZ managed the analyses of the study and wrote the first draft of the manuscript. Author AA was supervisor of this research and designed the study. Author A. Asrar designed the numerical analysis of the study and wrote the final draft of the manuscript.

Research Article

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ABSTRACT

In this research, we study some properties of compactons using Finite Element Method (FEM). This method is complicated for programming and very time consuming; but it is an accurate method. Using this method, we studied soliton properties and obtained results were acceptable. Then we studied compactons; Compactons are solitons with finite width or on the other hand solitons with no tail. This defined property for compactons was not observed in our simulation. It seems that breaking of compacton occurred regarding the entity of compacton equations, not by numerical error. In compactons- anti compactons collision, particle-like manner was not observed at all during this research. Perhaps it is due to suddenly vanishing of compactons on both ends.

Keywords: Soliton; finite element method; compacton; anti compacton.

1. INTRODUCTION

The localization of energy in space is a phenomenon which occurs in many physical phenomena. It is well known for about 50 years that solutions with this kind of characteristic arise in nonlinear dispersive equations in the form of solitons and more recently in the form of compactons [1-24]. Solitary waves were observed in 1834 for the first time. These waves

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have constant shape across time. It happens because of the balanced simultaneous effect of nonlinear and dispersive terms. Nonlinear term reduces the width of the wave shape and dispersive term makes it wide. Soliton is a solitary wave. One of the equations which has Soliton solution is Korteweg de Vries (KdV) equation. KdV is a special case of general partial differential equation k(m, n) [1],

$$u_t + (u^m)_x + (u^n)_{xxx} = 0 \qquad , m > 0\,,\ 1 < n \leq 3 \tag{1}$$

This equation has Compacton solutions for special values of m and n, e. g. $m = n = 2$ or $m = n = 3$. Compactons, by definition, have some characteristic properties of solitons such as particle-like elastic collision [2]. The shape of these waves remains unchanged after collision. Compctons have some basic differences with solitons too, such as

1. Compactons, unlike Solitons, have finite width [2].
2. Traveling velocity of Compactons, unlike Solitons, is independent of width [2].

Due to sudden vanishing shape of compactons on both ends, numerical study of these equations is difficult. Some numerical methods have been used for solving k(m, n) before e. g. Pseudo Spectral Method [2], Discontinuous Galerkin Method [3,4] and Finite Difference Method [5] and etc [1-24]. The numerical simulation of colliding solitary waves with compact support arising from the Rosenau–Hyman K(n, n) equation requires the addition of artificial dissipation for stability in the majority of methods. The price to pay is the appearance of trailing tails, amplitude damping, and delays as the solution evolves. These undesirable effects can be corrected by properly counterbalancing two sources of artificial dissipation; this procedure is designed by using the slow time evolution of the parameters of the solitary waves under the presence of the dissipation determined by means of adiabatic perturbation methods. The validity of the tail removal methodology is demonstrated on a Pad´e numerical scheme [19]. The tails are completely removed leaving only a small compact ripple at the original position of their front, and the numerical stability of the scheme under compacton collisions is preserved [19]. This paper has been extracted from simulation of k (m, n) equation with Finite Element Method (Galerkin method) by MATLAB without any additional terms for extra stability. This method is very complicated for programming and simulation [6].

2. SOLITONS AND COMPACTONS

2.1 Solitons

Some solutions of nonlinear KdV equations are solitons. KdV equation has been written as:

$$u_t - 6uu_x + u_{xxx} = 0 \tag{2}$$

t and x indices are time and space derivatives respectively. Balanced effect of nonlinear term uu_x and dispersive term u_{xxx} , cause initial wave shape to remain unchanged. Solution of KdV equation is a traveling wave with general form $u(x,t) = f(x - ct)$ and c is a constant showing wave velocity. Soliton solution of KdV equation is (Fig. 1)

$$u(x,t) = -\frac{1}{2}c.sech^2(x - ct) \tag{3}$$

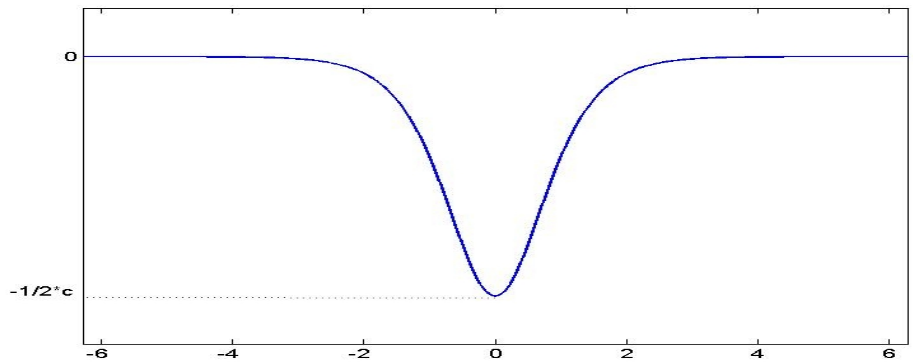

Fig. 1. Soliton solution of KdV equation

2.2 Compactons

k(m, n) equations were introduced for studying the role of dispersion in the waves. General form of these equations is very similar to KdV

$$u_t + (u^m)_x + (u^n)_{xxx} = 0 \qquad , m > 0\,, 1 < n \leq 3 \tag{4}$$

and t, x indices are time and space derivatives respectively. As a special case, k(m, n) equation for $m = 2$ and $n = 1$ is KdV equation. Characteristics property of solution of these equations is completely particle-like elastic collision. Unlike solitary waves with infinite width, these solutions have finite widths or on the other hand they have no tail [2]; so, they are compact and called compacton. In some articles, these equations were investigated for special values of m and n, and Compacton solution was extracted, e. g. for $m = n = 2$ and $m = n = 3$.

k(2, 2) equation is written as

$$u_t + (u^2)_x + (u^2)_{xxx} = 0 \tag{5}$$

and has closed form solution (Fig. 2)

$$u_c(x,t) = \begin{cases} \frac{4c}{3} cos^2\left(\frac{x-ct}{4}\right), & |x - ct| \leq 2\pi \\ 0\,, & otherwise \end{cases} \tag{6}$$

The invariance of Eq. (6) under the transformations $\begin{cases} u \to -u \\ t \to -t \end{cases}$, permits negative anti-compactons propagating in the opposite direction [1]. Because of their compact structure, neither compactons nor anti-compactons interact with each other until the moment of collision [7]. Eq. (6) has the following conserved quantities

$$\int u\, dx\,, \int u^3\, dx\,, \int u \cos x\, dx \text{ and } \int u \sin x\, dx \tag{7}$$

which have been investigated in ref. [1].

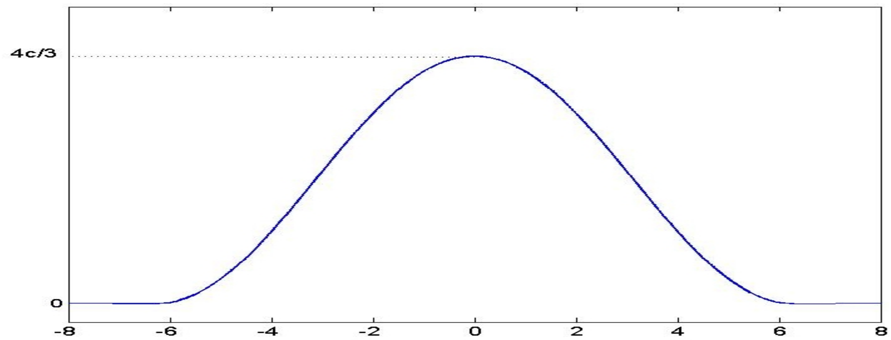

Fig. 2. k(2, 2) solution

k(3, 3) equation is written as

$$u_t + (u^3)_x + (u^3)_{xxx} = 0 \tag{8}$$

and has closed form solution (Fig. 3)

$$u_c(x,t) = \begin{cases} \pm\sqrt{\frac{3c}{2}}\cos\left(\frac{x-ct}{3}\right), & |x-ct| \le \frac{3\pi}{2} \\ 0, & otherwise \end{cases} \tag{9}$$

The solution given in Eq. (9) with the (+ sign) represents compacton and with the (- sign) represents anti-compacton. The K(3,3) has the conserved quantities

$$\int u\, dx\,, \int u^4\, dx \tag{10}$$

which have been investigated in ref. [1].

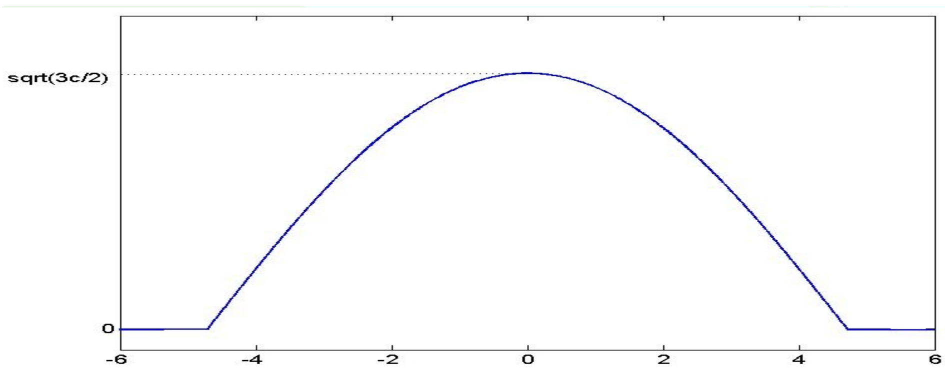

Fig. 3. k(3, 3) solution

3. THE FINITE ELEMENT METHODS

3.1 Introduction

The finite element method was first used by engineers to solve structural problems. They modeled a continuous structure using a number of "finite" (as distinct from infinitesimally small) elements that where connected at certain nodal points, and they required the forces to balance at each node. Only later was it realized that this technique good be thought of as a numerical method for solving the partial differential equations modeling the stresses in a continuous structure, and that similar methods good be used to solve other differential equations.

Now the "Finite Element Method" (really a class of methods) is generally considered to be a competitor of the finite difference methods and is used to solve as wide a range of ordinary and partial differential equations as the latter. Although finite element methods are usually substantially more difficult to program, this extra effort yields approximations that are of high-order accuracy even when a partial differential equation is solved in a general (nonrectangular) multidimensional region, and even when the solution varies more rapidly in certain portions of the region so that a uniform grid is not appropriate. These and other considerations have earned the finite element method great popularity in recent years both for initial value and (specially) boundary value differential equations [6].

In this method we divide space of problem into subspaces with the same sizes or, in most cases, different sizes. Then in each subspace, the solution of differential equation is approximated by the series of some arbitrary basic functions with unknown coefficients. We should try to find these unknown coefficients and consequently the solution.

3.2 The Galerkin Method [6]

The most widely used form of the finite element method is the "Galerkin" method. Although the Galerkin method can be applied to much more general problems, the following three-dimensional linear boundary value problems are chosen to make the analysis simple. We introduce the Galerkin method by studying this general problem

$$\begin{aligned} &\boldsymbol{\nabla}\cdot[D(x,y,z)\boldsymbol{\nabla u}] - a(x,y,z)u + f(x,y,z) = 0 && in\ R \\ &u = r(x,y,z) && on\ \ R_1 \\ &D\frac{\partial u}{\partial n} = -p(x,y,z)u + q(x,y,z) && on\ \ R_2 \end{aligned} \tag{11}$$

It is assumed that $D(x,y,z), a(x,y,z)$ and $f(x,y,z)$ are arbitrary functions of (x,y,z) and $D(x,y,z) > 0$.

Here $R = R_1 + R_2$ and $\partial u/\partial n$ represent the directional derivative of u in direction of the unit outward normal to the boundary. If the partial differential equation and second boundary equation are multiplied by a smooth function φ that is arbitrary except that it is required to satisfy $\varphi = 0$ on R_1 , and if these are integrated over R and R_2, respectively, then

$$\iiint\limits_R [\boldsymbol{\nabla}\cdot(D\boldsymbol{\nabla u}) - au + f]\,\varphi dxdydz + \iint\limits_{R_2}\left[-D\frac{\partial u}{\partial n} - pu + q\right]\varphi dxdy = 0 \tag{12.a}$$

Using $\varphi\nabla\cdot(D\nabla\boldsymbol{u}) = \nabla\cdot(\varphi D\nabla\boldsymbol{u}) - D(\nabla\boldsymbol{u}\cdot\nabla\boldsymbol{\varphi})$ and the divergence theorem:

$$\iiint\limits_R [-D\nabla\boldsymbol{u}.\nabla\varphi - au\varphi + f\varphi]\,dxdydz + \iint\limits_R [\varphi D\nabla\boldsymbol{u}\,\boldsymbol{n}]\,dxdy + \iint\limits_{R_2} [-\varphi D\nabla\boldsymbol{u}\,\boldsymbol{n} - pu\varphi + q\varphi]\,dxdy = 0 \quad (12.b)$$

The integrand in the first boundary integral is nonzero only on R_2, since $\varphi = 0$ on R_1. Thus

$$\iiint\limits_R [-D\nabla\boldsymbol{u}\cdot\nabla\varphi - au\varphi + f\varphi]dxdydz + \iint\limits_{R_2} [-pu\varphi + q\varphi]\,dxdy = 0 \tag{13}$$

Equation (13) is called the weak formulation of partial differential equation (11). It is almost equivalent to (11) in the sense that if u is smooth and satisfies (13) for any smooth φ vanishing on R_1, the steps leading from (11) to (13) can be reversed, so that u satisfies the partial differential equation and the second (natural) boundary condition. As part of either formulation is required to satisfy the first (essential) boundary condition. The Galerkin method attempts to find an approximate solution to the weak formulation (13) of the form

$$U(x,y,z) = \Omega(x,y,z) + \sum_{i=1}^{M} a_i\varphi_i\,(x,y,z) \tag{14}$$

where $\{\varphi_1\ ,\dots,\varphi_M\}$ is a set of linearly independent "trial" functions that vanishes on R_1 and Ω is another function that satisfy the essential boundary condition $\Omega = r$ on R_1. Clearly, U will satisfy $U = r$ on R_1 regardless of the values chosen for $a_1, \dots, a_M$.

It is impossible to find parameters a_i such that U satisfies (13) for arbitrary φ vanishing on R_1, since we only have a finite number of parameters. Thus, it is only required that (13) be satisfied for $\varphi = \varphi_1\ ,\dots,\varphi_M$ (each of which vanishes on R_1) :

$$\iiint\limits_R [-D\nabla\boldsymbol{U}\cdot\nabla\boldsymbol{\varphi_k} - aU\varphi_k + f\varphi_k]dxdydz + \iint\limits_{R_2} [-pU\varphi_k + q\varphi_k]\,dxdy = 0 \tag{15}$$

This can be written as a system of *M* linear equations for the *M* unknown parameters $a_1, \dots, a_M$.

$$\sum_{i=1}^{M} A_{ki}a_i = b_k \tag{16}$$

Where

$$b_k = \iiint\limits_R [f\varphi_k - D\nabla\boldsymbol{\Omega}\cdot\nabla\boldsymbol{\varphi_k} - a\Omega\varphi_k]\,dxdydz + \iint\limits_{R_2} [q\varphi_k - p\Omega\varphi_k]\,dxdy \tag{17}$$

$$A_{ki} = \iiint\limits_{R} [D\nabla\boldsymbol{\varphi_k} \cdot \nabla\boldsymbol{\varphi_i} + a\varphi_k\varphi_i]\, dxdydz + \iint\limits_{R_2} p\varphi_k\varphi_i dxdy \tag{18}$$

For example, two basic functions are "chapeau" function (20), (Fig. 4) and "cubic Hermit" function (21, 22) (Fig. 5):

chapeau function:

$$C_k(x) = \begin{cases} \dfrac{x - x_{k-1}}{x_k - x_{k-1}} & \text{for } x_{k-1} \le x \le x_k \\ \dfrac{x_{k+1} - x}{x_{k+1} - x_k} & \text{for } x_k < x \le x_{k+1} \\ 0 & \text{elsewhere} \end{cases} \tag{20}$$

where

$$x_{-1} < 0 = x_0 < x_1 < \cdots < x_N = 1 < x_{N+1}$$

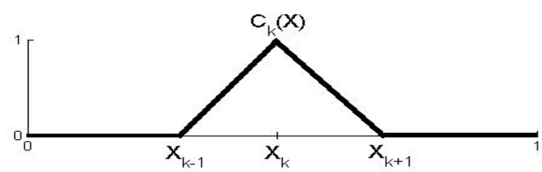

Fig. 4. "chapeau" function

and the cubic Hermit function:

$$H_k(x) = \begin{cases} 3\left[\dfrac{x - x_{k-1}}{x_k - x_{k-1}}\right]^2 - 2\left[\dfrac{x - x_{k-1}}{x_k - x_{k-1}}\right]^3 & \text{for } x_{k-1} \le x \le x_k \\ 3\left[\dfrac{x_{k+1} - x}{x_{k+1} - x_k}\right]^2 - 2\left[\dfrac{x_{k+1} - x}{x_{k+1} - x_k}\right]^3 & \text{for } x_k < x \le x_{k+1} \\ 0 & \text{elsewhere} \end{cases} \tag{21}$$

$$S_k(x) = \begin{cases} -\frac{(x-x_{k-1})^2}{(x_k-x_{k-1})} + \frac{(x-x_{k-1})^3}{(x_k-x_{k-1})^2} & \text{for } x_{k-1} \le x \le x_k \\ \frac{(x_{k+1}-x)^2}{(x_{k+1}-x_k)} - \frac{(x_{k+1}-x)^3}{(x_{k+1}-x_k)^2} & \text{for } x_k < x \le x_{k+1} \\ 0 & \text{elsewhere} \end{cases} \tag{22}$$

Where

$$x_{-1} < 0 = x_0 < x_1 < \cdots < x_N = 1 < x_{N+1}$$

It is obvious that

$$H_k(x_i) = \delta_{ki}, \qquad H_k^{'}(x_i) = 0$$
$$S_k(x_i) = 0, \qquad S_k^{'}(x_i) = \delta_{ki} \tag{23}$$

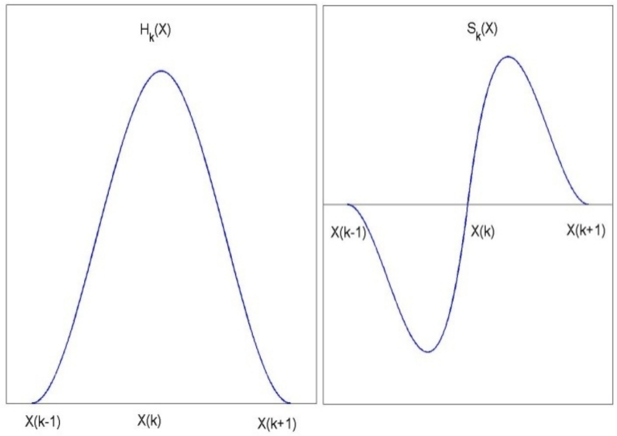

Fig. 5. Cubic hermit function

3.3 Time-Dependent Problems [6]

Many of advantages of the finite element method, such as its ability to accurately represent solution in general multidimensional domains, are still important when the problem is time dependent. The finite element method is therefore widely used to discrete the spatial variables in time-dependent problems. Consider, for example the general time-dependent problem:

$$c(x,y,z,t)u_t = \nabla.(D(x,y,z,t)\nabla \boldsymbol{u}) - a(x,y,z,t)u + f(x,y,z,t) \qquad in\ \ R$$

$$u = r(x,y,z,t) \qquad on\ R_1$$

$$D\,{}^{\partial u}/_{\partial n} = -p(x,y,z,t)u + q(x,y,z,t) \qquad on\ R_2$$

$$u = h(x,y,z) \qquad at\ \ t = 0 \tag{24}$$

It is assumed that $c > 0$ and ≥ 0 .

In a manner analogous to what we done for the corresponding steady state problem (13) , we find the weak formulation of (24) by multiplying the partial differential equation and second boundary condition by a smooth function $\varphi(x,y,z)$ that satisfies $\varphi = 0$ on R_1, and by integrating over R and R_2:

$$\iiint\limits_R cu_t\varphi dxdydz = \iiint\limits_R [\nabla.D\nabla\boldsymbol{u} - au + f]\varphi dxdydz + \iint\limits_{R_2}\left[-D\frac{\partial u}{\partial n} - pu + q\right]\varphi dxdy \tag{25}$$

Integrating by parts, remembering that $\varphi = 0$ on R_1, gives

$$\iiint\limits_R cu_t\varphi dxdydz = \iiint\limits_R [-D\nabla\boldsymbol{u}\,.\,\nabla\boldsymbol{\varphi} - au\varphi + f\varphi]dxdydz + \iint\limits_{R_2}[-pu\varphi + q\varphi]dxdy \tag{26}$$

If u is smooth and satisfies (26), for all t, for arbitrary smooth $\varphi(x,y,z)$ vanishing on R_1, then the steps from (24) to (26) are reversible, and therefore u satisfies the partial differential equation along with the second boundary condition. Thus if it is required that $u(x,y,z,t)$ satisfy the first boundary condition (on R_1) and the initial condition, in addition to being a smooth solution to the weak formulation (26), u will be a solution to the partial differential equation (24).

In the continuous-time Galerkin method, we attempt to find a solution to the weak formulation (26) of the form

$$U(x,y,z,t) = \Omega(x,y,z,t) + \sum_{i=1}^{M} a_i(t)\,\varphi_i(x,y,z) \tag{27}$$

where $\{\varphi_1 \ ,\dots,\varphi_M\}$ is a set of linearly independent functions that vanishes on R_1 ,and Ω is another function that satisfies the essential boundary condition satisfy $U = r$ on R_1. Clearly, U will satisfy the essential boundary condition regardless of how the coefficients $a_i(t)$ are chosen.

As in the steady state case, it is not possible to find coefficients such U that satisfies (26) for arbitrary φ vanishing on ∂R_1, so it is only required (26) to be satisfied for $\varphi = \varphi_1 \ ,\dots,\varphi_M$:

$$\iiint\limits_R cU_t\varphi_k dxdydz = \iiint\limits_R [-D\nabla \boldsymbol{U}.\nabla\boldsymbol{\varphi_k} - aU\varphi_k + f\varphi_k]\,dxdydz + \iint\limits_{R_2} [-pU\varphi_k + q\varphi_k]\,dxdy \tag{28}$$

Substituting (27) for U in (28) gives

$$\sum_{i=1}^{M} B_{ki}(t)a_i^{'}(t) = -\sum_{i=1}^{M} A_{ki}(t)a_i(t) + b_k(t) \tag{29}$$

where

$$B_{ki}(t) = \iiint\limits_R c\varphi_k\varphi_i dxdydz$$

$$A_{ki}(t) = \iiint\limits_R [D\nabla\boldsymbol{\varphi_k}.\nabla\boldsymbol{\varphi_i} + a\varphi_k\varphi_i]dxdydz + \iint\limits_{R_2} p\varphi_k\varphi_i dxdy$$

$$b_k(t) = \iiint\limits_R \left[-c\Omega_t\varphi_k - D\nabla\boldsymbol{\Omega}.\nabla\boldsymbol{\varphi_k} - a\Omega\varphi_k + f\varphi_k\right]dxdydz + \iint\limits_{R_2} [-p\Omega\varphi_k + q\varphi_k]dxdy \tag{30}$$

In the steady state case, the Galerkin method led to a system of algebraic equation for the unknown coefficients a_i. Here it leads to a system of ordinary differential equations:

$$B(t)\boldsymbol{a}^{'} = -A(t)\boldsymbol{a} + \boldsymbol{b}(t) \tag{31}$$

for the unknown coefficient functions $a_i(t)$.

The initial values for this ordinary differential equation system are obtained by requiring that, at $t = 0$, U approximately satisfy initial condition in (24). We followed the above general approaches for KdV, k(2, 2) and k(3, 3) equations. Details of computation were omitted for abbreviation.

4. CONCLUSION

In one part of our research, we simulate the KdV equation. It obtained soliton travelling with constant shape (Fig. 6), collision, and then separation (Fig. 7).

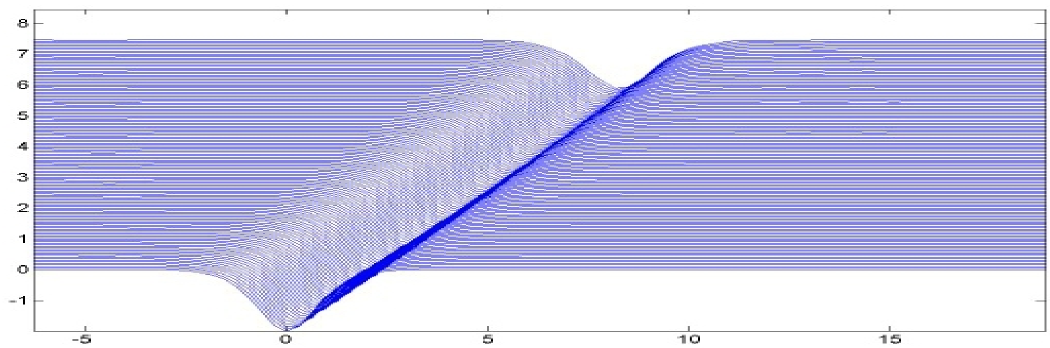

Fig. 6. Soliton moves without change in shape

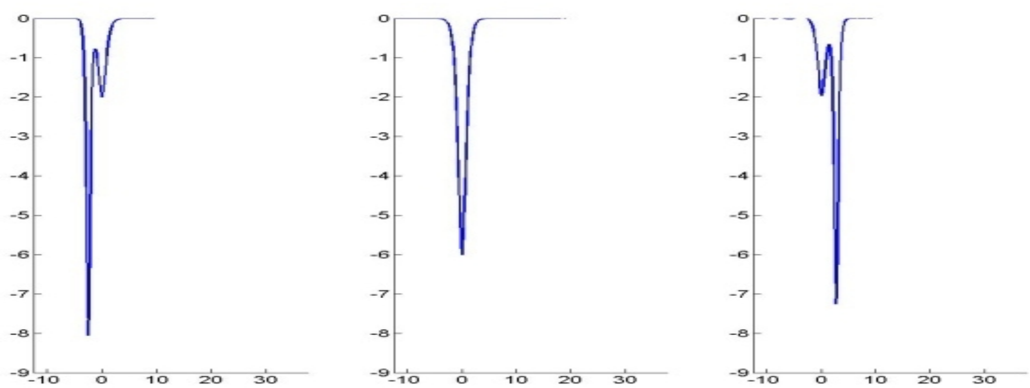

Fig. 7. Collision of two solitons and separation

We saw that if we solve KdV equation with arbitrary initial shape (32),

$$u(x) = -2.5sech^2x \tag{32}$$

some perturbations leave the shape and soliton solution $u(x) = -2sech^2x$ is appeared and travels without change (Fig. 8). For comparing between extracted soliton from arbitrary initial shape and closed form solution of KdV, we insert equation (3) by dots in (Fig. 8) at related time.

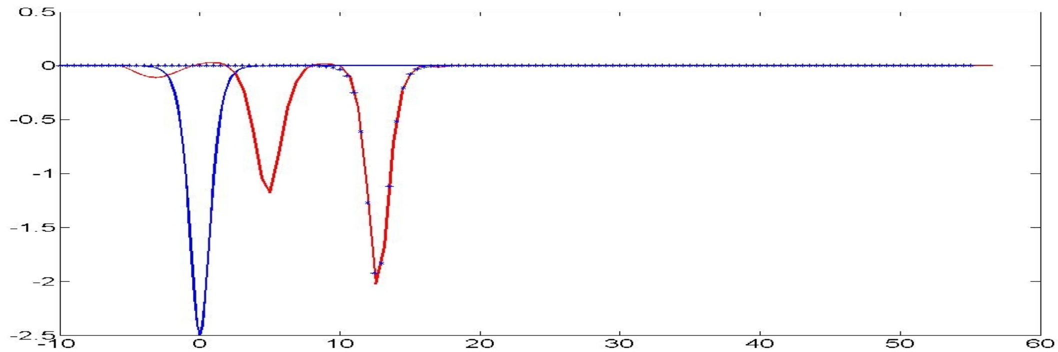

Fig. 8. Some perturbations leave initial arbitrary shape (blue) and the soliton moves without change in shape (red shape in right), dots on right valley show the soliton shape coincidence

But for the compactons, even with closed form solution of k(2, 2) and k(3, 3) as initial wave shapes, in the time evolution of related equations, some perturbations appear and then blow up. We investigated k(2, 2) and k(3, 3) by Finite Element Method with a wide variety of basic functions, space step sizes and time step sizes. This happens even with reducing space and time step sizes for long time simulation for all basic functions (Figs. 9, 10).

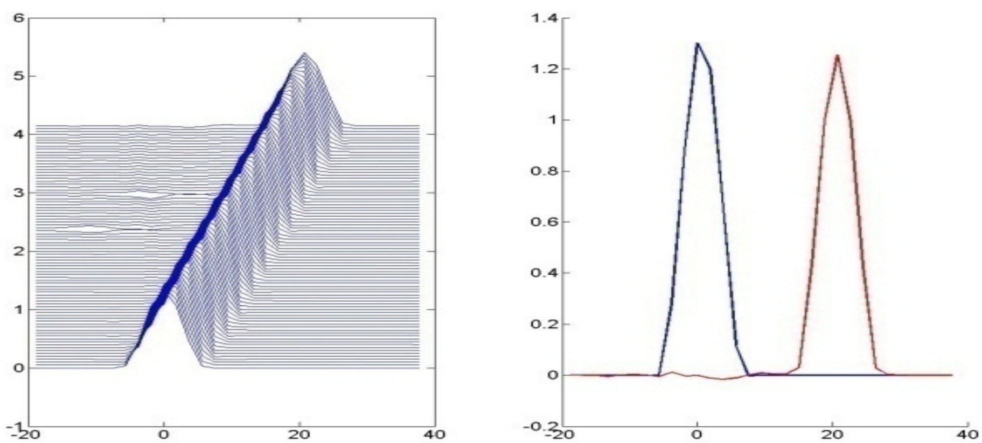

Fig. 9. Movement of k(2, 2) compacton before crashing

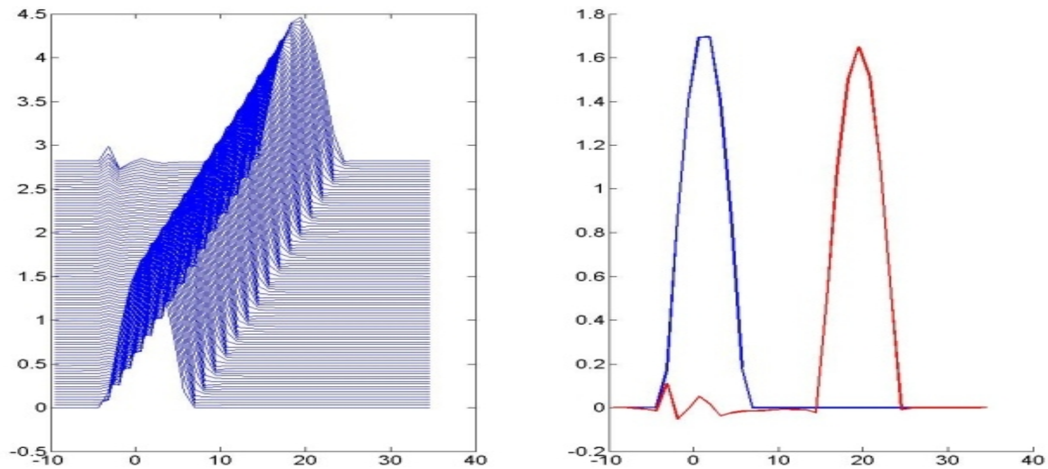

Fig. 10. Movement of k(3, 3) compacton before crashing

We saw compacton and anti compacton collision for k(2, 2) and then for k(3, 3), but exactly after separation, some perturbations appear and then blow up (Fig. 11, 12).

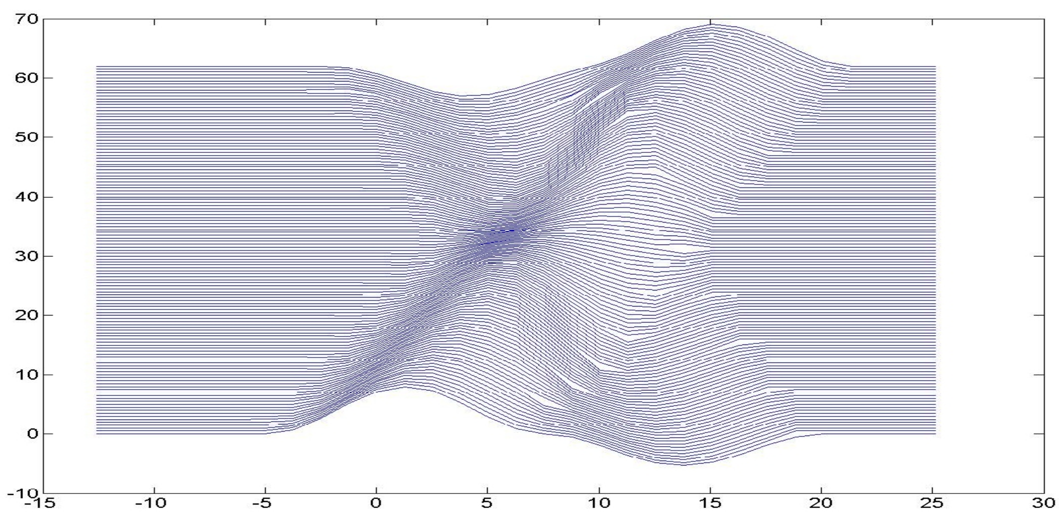

Fig. 11. Collision of k(2, 2) compacton and anti compacton

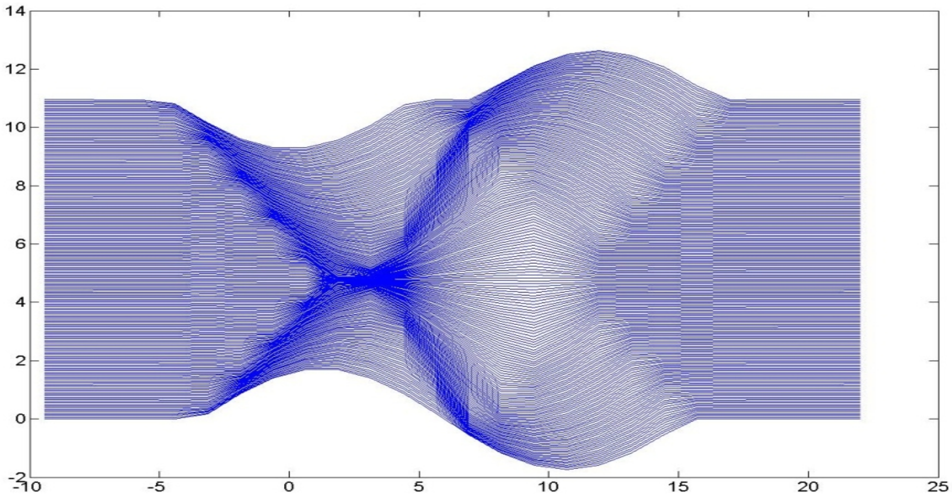

Fig. 12. Collision of k(3, 3) compacton and anti compacton

Not only in Finite Element Method, but also for all frequently used numerical methods, the compacton has not time evolution and particle-like collision. Perhaps, main role in divergence is caused by sudden vanishing of the Compactons on both ends. This event causes discontinuity in derivatives. Is it true that the main part or all of divergence is caused by numerical method? We should answer to this question carefully. All of numerical methods have some round off or truncation errors. But it is accepted that numerical methods are applicable, specifically for the problems with no analytic or closed form solution. One of the most accurate numerical methods is Finite Element Method and we obtained some acceptable results for solitons by this method. So, perhaps some of properties that enumerated for compactons are unreal. Can we confine a wave in this limit and relate particle like manner to it?

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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