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Boundary Layer Slip and Heat Transfer in Stagnation Point Flow over a Steady Stretching Surface Embedded in a Porous Medium with Heat Sources

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Authors' contributions

This work was carried out in collaboration between all authors. Author MF designed the study, wrote the protocol. Authors DL and MF carried out all numerical work and performed the mathematical analysis. Author MF wrote the first draft of the manuscript. Authors DL and BRR managed the literature searches and edited the manuscript. All authors read and approved the final manuscript.

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Abstract

The aim of this work is to study the 2-D boundary layer stagnation point flow from a stretching surface embedded in a porous medium with velocity/thermal slip conditions on the wall and heat generation or absorption. Under boundary layer approximations, the governing boundary layer equations of the problem can be reduced to two ordinary differential equations and solved by using Maple code for some values of the problem parameters. The influences of problem parameters on the flow characteristics such as velocity and temperature profiles, skin friction and the Nusselt number are clearly discussed graphically and in tables. It is found that the flow field is influenced greatly by the ratio of the free stream velocity (u_e) and the stretching velocity (u_w) as well as the velocity/thermal slip parameter. We have compared our results with open literature and found excellence agreement.

Keywords: Velocity slip parameter; thermal slip parameter; stagnation point; stretching surface; porous medium.

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1 Introduction

Boundary layer flow over s stretching surface has been studying due to its applications in science and engineering specially industry. Crane [1] developed similarity solutions of continuous stretching sheet where the velocity proportional to the distance from a stagnation point. Boundary layer flow through porous media over a stretching sheet with internal heat generation/absorption and suction/blow has been studies by [2,3]. Mahapatra [4] also initiated the boundary layer approximation in the stagnation point flow towards a stretching sheet. In the above studies they do not consider slip flow on the wall. On the other hand, boundary layer flow with slip effects over stretching sheet has been reported by [5,6].

Nadeem et al. [7] have studied the axisymmetric stagnation flow of a micropolar nanofluid in a moving cylinder analytically by homotopy analysis technique. Boundary layer stagnation point flow and heat transfer of nanoparticles over an exponentially stretching surface in a porous medium has been discussed by [8]. Morever, steady mixed convective boundary layer and heat transfer of stagnation point flow were studied by [9] using keller box method. On the other hand [10] have considered the stagnation point flow over an exponentially stretching sheet of a second grade fluid through homotopy analysis and Keller box methods.

In light of the above, the objective of the present paper is to study heat transfer boundary layer stagnation point flow from a stretching surface based on velocity/ thermal slip on the wall in a fluid saturated porous medium considering heat generation or absorption.

2 Mathematical Formulation and Discussion

Consider the steady two dimensional flow of a viscous and incompressible fluid near the stagnation point on a stretching surface placed in a plane y = 0 embedded in a fluid saturated porous medium. The stretching surface has a uniform temperature T_w and a linear velocity u_w, while the velocity of the flow external to the boundary layer is $u_e(x)$. Also, the effects of heat source are included in the energy equation. Governing equation of the present problem can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+u_e\frac{du_e}{dx}-\left(\frac{\upsilon}{k}\right)(u_e-u) \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-\frac{Q}{\rho c_p}(T-T_\infty) \tag{3}$$

The boundary conditions are

$$\begin{aligned} &u=u_w=cx+N_1\upsilon\frac{\partial u}{\partial y},\, v=0,\, T=T_{\mathrm{w}}+D_1\frac{\partial T}{\partial y} \qquad \text{at} \quad y=0 \\ &u=u_e(x)=ax,\, T=T_\infty \qquad \text{as} \quad y\to\infty \end{aligned} \tag{4}$$

Where N_1 and D_1 is the velocity and thermal slip factor respectively.

To facilitate the analysis, we introduce the following dimensionless variables:

$$\eta = y\left(\frac{c}{\upsilon}\right)^{1/2},\ u = cxf'(\eta), v = -(c\upsilon)^{1/2} f(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

Substituting (6) into (2)-(3), we have

$$f''' + ff'' - (f')^2 + R\left(\frac{a}{c} - f'\right) + \frac{a^2}{c^2} = 0 \tag{6}$$

$$\theta'' + \Pr(f\,\theta' + \lambda\theta) = 0 \tag{7}$$

$$\begin{aligned} &f = 0,\ \ f' = 1 + \alpha f'',\ \ \theta = 1 + \beta\theta'\ at\ \eta = 0 \\ &f' = a/c,\ \ \theta = 0, at\ \eta = \infty \end{aligned} \tag{8}$$

Where, $R = \dfrac{\upsilon}{ck}$ is the permeability parameter, $\lambda = \dfrac{Q_0}{c\rho c_p}$ is the heat generation or absorption, $\alpha = N_1\sqrt{c\upsilon}$ is the velocity slip parameter and $\beta = D_1\sqrt{c/\upsilon}$ is the thermal slip parameter.

The two sets of boundary value problems (6), (7) and (8) were solved by using the *dsolve* routine from MAPLE [11]. Table 1 show the comparison results against the results reported by Mahapatra [4] and the agreement is excellent.

Table 1. Comparison of results for $-\theta'(0)$ when $R = 0, \alpha = 0, \beta = 0, \lambda = 0$

a/c	**Pr = 1**		**Pr = 1.5**	
	Mahaparta and Gupta 2002	**Present**	**Mahaparta and Gupta 2002**	**Present**
0.1	0.603	0.60215742	0.777	0.77680103
0.2	0.625	0.62446886	0.797	0.79712229
0.5	0.692	0.69244941	0.863	0.86479409
1	0.796	0.79788456	0.974	0.97720507
2	0.974	0.97872695	1.171	1.17809903
3	1.124	1.13209179	1.341	1.35194366

Table 2. Values of $f''(0)$ and $-\theta'(0)$ for different values of α, R and a/c when $\Pr = 0.7, \beta = 0.5, \lambda = 0.1$

R	a/c	$f''(0)$			$-\theta'(0)$		
		$\alpha = 0.1$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 0.1$	$\alpha = 0.5$	$\alpha = 1$
0.5	0.1	-0.998243	-0.657759	-0.469132	0.284208	0.232737	0.194685
	0.5	-0.646936	-0.417954	-0.292226	0.393729	0.375760	0.364828
	1.5	0.823376	0.501814	0.335299	0.532773	0.544384	0.550024
1	0.1	-1.125256	-0.721802	-0.505852	0.262492	0.205533	0.165675
	0.5	-0.705458	-0.443735	-0.305349	0.389880	0.371818	0.361363
	1.5	0.865748	0.516377	0.341534	0.533924	0.545342	0.550733
1.5	0.1	-1.235065	-0.773005	-0.533485	0.244811	0.184560	0.144662
	0.5	-0.757713	-0.465403	-0.315959	0.386672	0.368667	0.358686
	1.5	0.905041	0.529401	0.347022	0.534946	0.546175	0.551342

In Tables 2 and 3 we compute the local skin-friction and local Nusselt number along the surface for different values of the parameters.

Table 3. Values of $\theta(0)$ and $-\theta'(0)$ for different values of $\Pr, \lambda$ and β when $R=0.5, \alpha=0.5$ a/c=2

Pr	λ	$f''(0)$			$-\theta'(0)$		
		$\beta=0.1$	$\beta=0.5$	$\beta=1$	$\beta=0.1$	$\beta=0.5$	$\beta=1$
0.1	0.1	0.967605	0.856605	0.749177	0.323952	0.286789	0.250823
	0.2	0.968825	0.861406	0.756553	0.311753	0.277188	0.243447
	0.3	0.970081	0.866396	0.764284	0.299187	0.267209	0.235716
0.7	0.1	0.920840	0.699387	0.537736	0.791599	0.601227	0.462264
	0.2	0.923839	0.708117	0.548128	0.761605	0.583765	0.451872
	0.3	0.926945	0.717329	0.559246	0.730546	0.565342	0.440754
7	0.1	0.793256	0.434191	0.277295	2.067437	1.131618	0.722704
	0.2	0.800677	0.445492	0.286581	1.993221	1.109016	0.713419
	0.3	0.808480	0.457783	0.296834	1.915198	1.084435	0.703166

From these we have seen that:

1) The local skin friction and local Nusselt number decreases with the increase in R and β.
2) The local skin friction and local Nusselt number increases with the increase in a/c.
3) The local skin friction increase and local Nusselt number decrease with the increase in α and λ.
4) The local skin friction decrease and local Nusselt number increase with the increase in Pr.

Fig. 1, show the dimensionless velocity (left) and temperature (right) for a/c. It is seen that the thickness of the hydrodynamic and thermal boundary layer increases and decreases respectively with increasing a/c. This indicate that the fluid motion near the stagnation point flow increases and decreases.

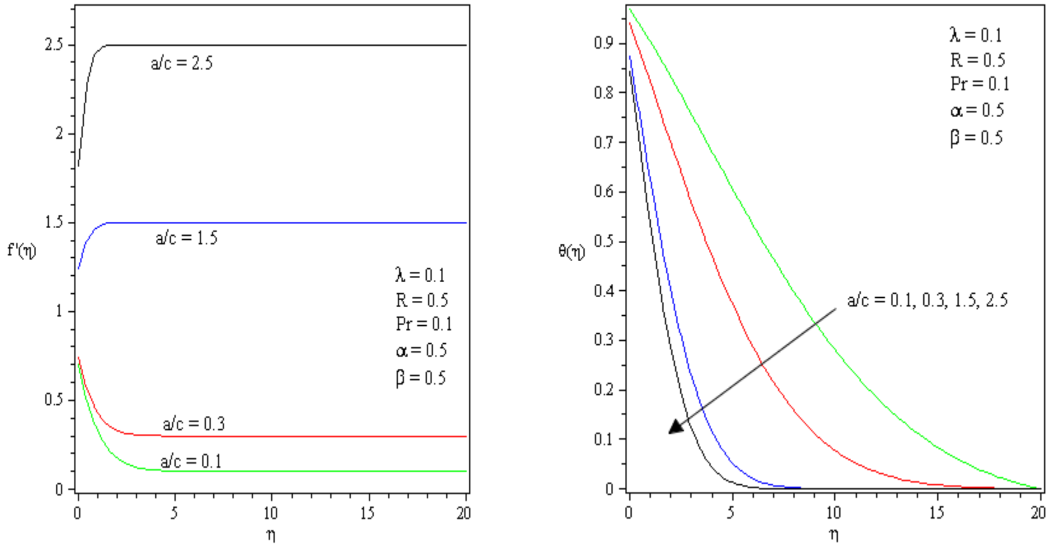

Fig. 1. Dimensionless flow profiles

Fig. 2, show the dimensionless velocity and temperature for different values of porous parameter R. It is seen that the boundary layer reduced the fluid motion by this parameter. This behavior is due to the fact that near the surface of the sheet, where the viscosity effects are dominant increase in porous parameter corresponds to lower viscosity and hence lower profile near the surface.

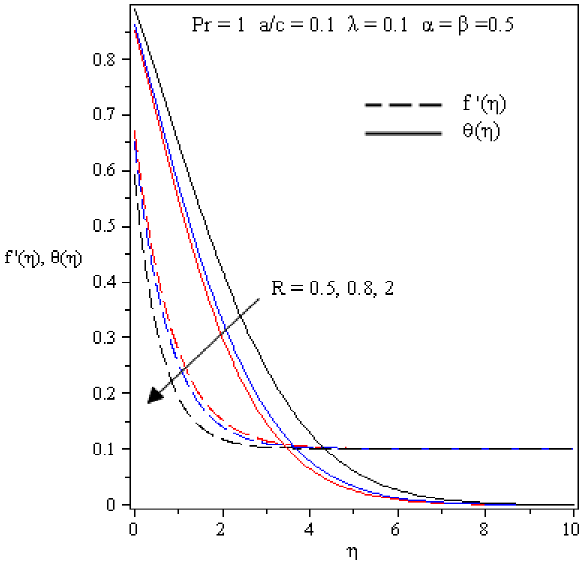

Fig. 2. Dimensionless flow profiles

Fig. 3, show the dimensionless velocity and temperature for velocity slip parameter. It is seen that the thickness of the hydrodynamic and thermal boundary layer decreases and increases respectively with increasing α .

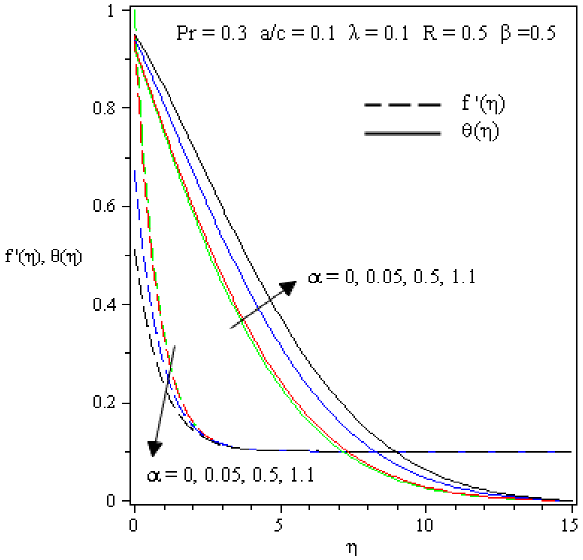

Fig. 3. Dimensionless flow profiles

Fig. 4, show the dimensionless velocity and temperature for thermal slip parameter. It is seen that the thickness of the thermal boundary layer decreases with increasing β. We have seen almost negligible impact on hydrodynamic boundary as we expect.

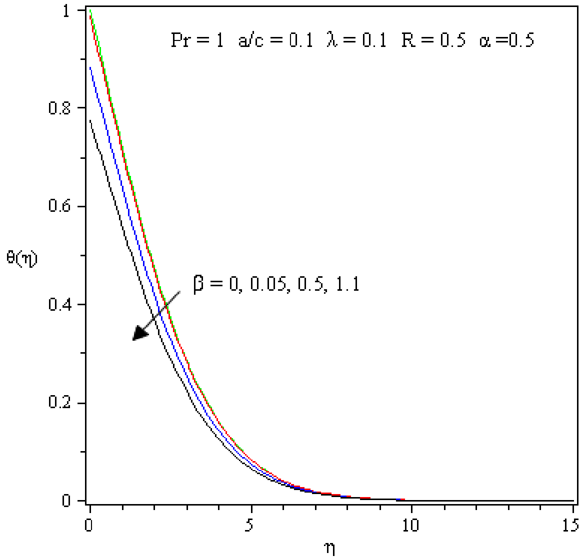

Fig. 4. Dimensionless flow profiles

Fig. 5, show the dimensionless temperature profiles for Prandtl number. It is seen that the thickness of the thermal boundary layer decreases with increasing Pr. Lower Pr induced more flow than in the case of Higher Pr. Larger Prandtl number fluids have higher viscous diffusivity and weaker thermal diffusivity.

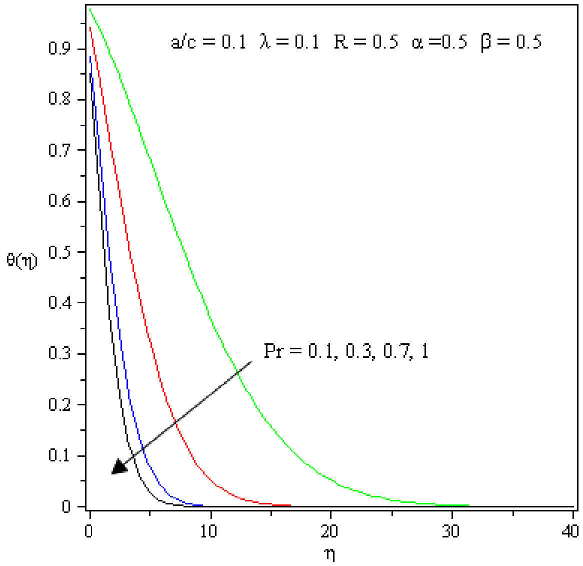

Fig. 5. Dimensionless flow profiles

Fig. 6, show the dimensionless temperature profiles for heat generation/absorption parameter. It is seen that the thickness of the thermal boundary layer increases with increasing λ. Note that $\lambda \prec 0$ and $\lambda \succ 0$ indicate absorption and heat generation respectively.

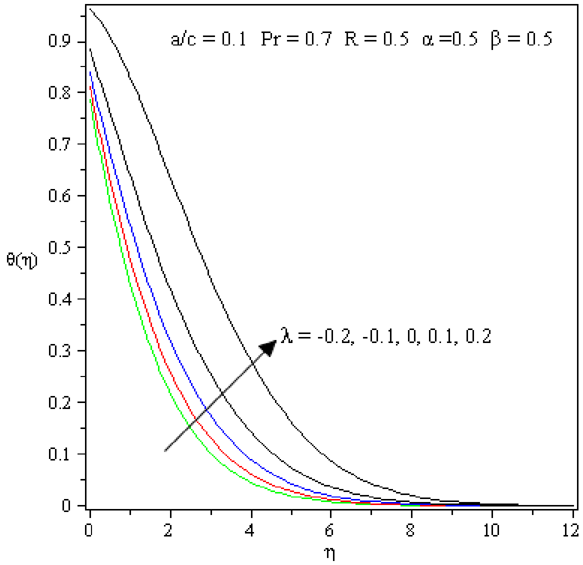

Fig. 6. Dimensionless flow profiles

3 Conclusions

2-D boundary layer stagnation point flow from a stretching surface embedded in a porous medium with velocity/thermal slip conditions on the wall and heat generation or absorption is studied. Velocity and temperature flow profiles as well physical parameters such as skin friction and Nusselt number are presented. Our results are similar as those presented in literature. We observe significant impacts of flow phenomena considered in this study.

Competing Interests

Authors have declared that no competing interests exist.

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