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Intuitionistic Fuzzy Nano Generalized Closed Sets

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Authors' contributions

This work was carried out in collaboration between both authors. Author ASAR designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author MR managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper we introduce and study the notion of intuitionistic fuzzy nano generalized closed sets in intuitionistic fuzzy nano topological spaces and some of its properties and results. A real time application of intuitionistic fuzzy nano topology under intuitionistic fuzzy nano upper approximation in multi criterion decision making is discussed with an example.

Keywords: Intuitionistic fuzzy nano closed sets; intuitionistic fuzzy nano generalized closed sets; multi criterion decision making.

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1 Introduction

Levine N. in [1] introduced the notion and decomposition of continuity in topological spaces. Jingcheng Tong in [2] introduced the notion of A-sets and A-continuity and established a decomposition of continuity.

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Further, Jingcheng Tong in [3] introduced the notion of B-sets and B-continuity and established a decomposition of continuity. Ganster. M and Reilly I. L. in [4] improved Tong's decomposition result. Jingcheng Tong in [5] generalized. Levine [6] introduced generalized closed sets. Lellis Thivagar M. and Carmel Richard in [7] introduced the notion of Nano topology which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. Lellis Thivagar M and Carmel Richard in [7,8] studied a new class of functions called nano continuous functions and their characterizations in nano topological spaces. Bhuvaneswari, K. [9] studied generalized closed sets in nano topological spaces. Stephan Antony Raj A and Ramachandran M [10] introduced the notion of intuitionistic fuzzy nano topological spaces.

2 Preliminaries

Definition 2.1 [7]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 1.2, $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$.
(2) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano-open sets. If $(U, \tau_R(X))$ is a nano topological space [8] where $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by $NInt(A)$. $NInt(A)$ is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $NCl(A)$. That is, $NCl(A)$ is the smallest nano closed set containing A.

Definition 2.2 [7]: Let $(U, \tau_R(X))$ and $(V, \tau_R^{'}(Y))$ be two nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ is nano continuous on U if the inverse image of every nano-open set in V is nano-open in U.

Definition 2.3 [9]: Let $(U, \tau_R(X))$ be a nano topological space. A subset A of $(U, \tau_R(X))$ is called nano generalized closed set (briefly Ng- closed) if $NCl(A) \subseteq$ V where $A \subseteq V$ and V is nano open.

Definition 2.2 [10]: Let U be a non-empty finite set of objects called the universe and R be an intuitionistic fuzzy equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the intuitionistic fuzzy approximation space(In short IFAS). Let$X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $IFL_R(X)$. That is, $IFL_R(X) = \bigcup_{x \in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $IFU_R(X)$. That is, $IFU_R(X) = \bigcup_{x \in U}\{R(x): R(x) \cap X \neq \emptyset\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $IFB_R(X)$. That is,

   $IFB_R(X) = IFU_R(X) - IFL_R(X)$.

Let U be the universe, R be an intuitionistic fuzzy equivalence relation on U and $\tau_R(X) = \{1 \sim, 0 \sim, IFL_R(X), IFU_R(X), IFB_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) $1\sim$ and $0\sim\in\tau_R(X)$.
(2) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the intuitionistic fuzzy nano topology on U with respect to X. We call $(U,\tau_R(X))$ as the intuitionistic fuzzy nano topological space. The elements of $\tau_R(X)$ are called as intuitionistic fuzzy nano-open sets. If $(U,\tau_R(X))$ is a intuitionistic fuzzy nano topological space(In short IFNTS) where $X\subseteq U$ and if $A\subseteq U$, then the intuitionistic fuzzy nano interior of A is defined as the union of all intuitionistic fuzzy nano-open subsets of A and it is denoted by $IFNInt(A)$. $IFNInt(A)$ is the largest intuitionistic fuzzy nano-open subset of A. The intuitionistic fuzzy nano closure of A is defined as the intersection of all intuitionistic fuzzy nano closed sets(In short IFNCS) containing A and it is denoted by $IFNCl(A)$. That is, $IFNCl(A)$ is the smallest intuitionistic fuzzy nano closed set containing A.

3 Intuitionistic Fuzzy Nano Generalized Closed Set

Throughout this paper $(U,\tau_R(X))$ is an intuitionistic fuzzy nano topological space with respect to X where X ⊆ U , R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R.

Definition 3.1: Let $(U,\tau_R(X))$ be an intuitionistic fuzzy nano topological space. A subset A of $(U,\tau_R(X))$ is called intuitionistic fuzzy nano generalized closed set (briefly IFNg- closed) if *IFNCl(A)* ⊆ V where A ⊆ V and V is intuitionistic fuzzy nano open(In short IFNOS).

Example 3.2: Let (*U,R*) be an intuitionistic fuzzy approximation space(In short *IFAS*) where $U = \{a_1, a_2, a_3\}$ with $R = \{\langle(a_1, a_1),1,0\rangle,\langle(a_1,a_2),0.5,0.5\rangle,\langle(a_2,a_1),0.5,0.5\rangle,\langle(a_2,a_2),1,0\rangle,$
$\langle(a_2, a_3),0.3,0.7\rangle,\langle(a_3,a_2),0.3,0.7\rangle,\langle(a_3,a_3),1,0\rangle,\langle(a_1,a_3),0.4,0.6\rangle,\langle(a_3,a_1),0.4,0.6\rangle\}$.

Let $A = \{\langle a_1,0.7,0.3\rangle,\langle a_2,0.5,0.5\rangle,\langle a_3,0.6,0.3\rangle\}$ be an intuitionistic fuzzy set(In short *IFS*)on *U* then $\tau_R(X)=\{1\sim,0\sim,\{\langle a_1,0.7,0.3\rangle,\langle a_2,0.5,0.5\rangle,\langle a_3,0.6,0.3\rangle\},\{\langle a_1,0.5,0.5\rangle,\langle a_2,0.5,0.5\rangle,\langle a_3,0.6,0.3\rangle\},\{\langle a_1,0.5,0.5\rangle,$

$\langle a_2,0.5,0.5\rangle,\langle a_3,0.3,0.6\rangle\}\}$. Let $V = \{\langle a_1,0.7,0.3\rangle,\langle a_2,0.5,0.5\rangle,\langle a_3,0.6,0.3\rangle\}$ be an *IFS* in U

and let $B = \{\langle a_1,0.2,0.7\rangle,\langle a_2,0.5,0.5\rangle,\langle a_3,0.4,0.3\rangle\}$ be an *IFS* such that $B\subseteq V$ then *IFNCl (B)* = $\{\langle a_1,0.5,0.5\rangle,\langle a_2,0.5,0.5\rangle,\langle a_3,0.6,0.3\rangle\}\subseteq V$. That is B is said to be IFNg- closed in $(U,\tau_R(X))$.

Theorem 3.3: A subset B of $(U,\tau_R(X))$ is IFNg–closed if *IFNCl(B) – B* contains no nonempty IFNg-closed set.

Proof: Suppose if B is IFNg-closed. Then *IFNCl(B)* ⊆ V where *B* ⊆ *V* and V is IFNO. Let Y be a intuitionistic fuzzy nano closed subset of *IFNCl(B) – B* . Then $B\subseteq Y^c$ and Y^c is IFNO. Since B is Ng – closed, *IFNCl(B)* ⊆ Y^c or Y ⊆ $(IFNCl(B))^c$. That is, Y ⊆ *IFNCl(B)* and Y ⊆ $(IFNCl(B))^c$ implies that Y⊆ ø . So Y is empty.

Theorem 3.4: If A and B are IFNg-closed, then *A U B* is IFNg-closed.

Proof: Let A and B are IFNg- closed sets. Then *IFNCl(A)* ⊆ *V* where *A* ⊆ *V* and *V* is IFNO and *IFNCl(B)* ⊆ *V* where B ⊆ V and V is IFNOS . Since A and B are subsets of V, *A U B* is a subset of V and V is IFNOS. Then *IFNCl(A U B) = IFNCl(A) U IFNCl(B)* ⊆ *V* which implies that *AUB* is IFNg-closed.

Theorem 3.5: If C is IFNg – closed and C ⊆ D ⊆ *IFNCl(C)* , then D is IFNg-closed.

Proof: Let D ⊆ V where V is IFNOS in $\tau_R(X)$. Then C⊆ D implies C ⊆ V. Since C in IFNg-closed, *IFNCl(A)* ⊆ V. Also *C* ⊆ *NCl(D)* implies *IFNCl(D)* ⊆ *IFNCl(C)*. Thus *IFNCl(D)* ⊆ *V* and so D is IFNg-closed.

Theorem 3.6: Every IFNCS is a IFNg-closed set.

Proof: Let B $\subseteq$ V and V is IFNOS in $\tau_R(X)$. Since B is IFNCS, *IFNCl(B)* $\subseteq$ B. That is *IFNCl(B)* $\subseteq$ B $\subseteq$ V. Hence B is a IFNg-closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7: Let (U,R) be an *IFAS* where $U = \{u,v,w\}$ with

R=$\{\langle(u,u),1,0\rangle,\langle(u,v),0.2,0.2\rangle,\langle(v,u),0.2,0.2\rangle,\langle(v,v),1,0\rangle,\langle(v,w),0.3,0.3\rangle,\langle(w,v),0.3,0.3\rangle,$ $\langle(w,w),1,0\rangle,\langle(u,w),0.1,0.2\rangle,\langle(w,u),0.1,0.2\rangle\}$.

Let B = $\{\langle u,0.5,0.3\rangle,\langle v,0.5,0.4\rangle,\langle w,0.4,0.3\rangle\}$ be an intuitionistic fuzzy set(In short *IFS*)on U then

$\tau_R(X)$=$\{1\sim,0\sim,\{\langle u,0.5,0.3\rangle,\langle v,0.5,0.3\rangle,\langle w,0.4,0.3\rangle\},\{\langle u,0.4,0.3\rangle,\langle v,0.4,0.4\rangle,\langle w,0.4,0.3\rangle\},\{\langle u,0.3,0.4\rangle,$

$\langle v,0.4,0.4\rangle,\langle w,0.3,0.4\rangle\}\}$. Let V = $\{\langle u,0.5,0.3\rangle,\langle v,0.5,0.3\rangle,\langle a_3,0.4,0.3\rangle\}$ be an *IFS* in U and

let B = $\{\langle u,0.3,0.5\rangle,\langle v,0.4,0.4\rangle,\langle w,0.2,0.4\rangle\}$ be an *IFS* such that $B \subseteq V$,

IFNCl(B) = $\{\langle a_1,0.5,0.5\rangle, \langle a_2,0.5,0.5\rangle, \langle a_3,0.3,0.6\rangle\} \subseteq V$. That is B is said to be IFNg- closed in $(U,\tau_R(X))$. But B is not IFNCS.

Theorem 3.8: An IFNg- closed set A is IFNCS if and only if *IFNCl(A) – A* is IFNCS.

Proof: (Necessity) Let A is IFNCS. Then *IFNCl(A) = A* and so *IFNCl(A) – A = ø* which is IFNCS.

(Sufficiency) Suppose *IFNCl(A) – A* is IFNCS. Then *IFNCl(A) – A = ø* since A is IFNCS. That is, *IFNCl(A) = A* or A is IFNCS.

Theorem 3.9: Suppose that $B \subseteq A \subseteq U$, B is an IFNg-closed set relative to A and that A is an IFNg-closed subset of U. Then B is IFNg-closed relative to U.

Proof: Let $B \subseteq V$ and suppose that V is IFNOS in U. Then $B \subseteq A \cap V$. Therefore *IFNCl(B)*$\subseteq A \cap V$. It follows that $A \cap$*IFNCl(B)*$\subseteq A \cap V$ and $A \subseteq V \cup$*IFNCl(B)*. Since A is IFNg- closed in U, we have *IFNCl(A)* $\subseteq V \cup$*IFNCl(B)*. Therefore *IFNCl(B)*$\subseteq$ *IFNCl(A)*$\subseteq V \cup$*IFNCl(B)* and so *IFNCl(B)*□ *V*. Then B is IFNg-closed relative to V.

Corollary 3.10: Let A be a IFNg-closed set and suppose that F is a IFNCS. Then A∩F is an IFNg-closed set.

Theorem 3.11: For each a∈U, either {a} is IFNCS (or) $\{a\}^C$ is IFNg- closed in $\tau_R(X)$.

Proof: Suppose {a} is not IFNCS in U. Then $\{a\}^C$ is not IFNOS and the only IFNOS containing $\{a\}^C$ is $V \subseteq U$. That is $\{a\}^C \subseteq U$. Therefore *IFNCl*$(\{a\}^C) \subseteq U$ which implies $\{a\}^C$ is IFNg-closed set in $\tau_R(X)$.

Theorem 3.12: Let A be a g-closed subset of (X, τ). Then Cl(A) − A does not contain any non-empty closed sets.

Proof: Let F ∈ C (X) such that F ⊆ Cl(A) − A. Since X − F is open, A ⊆ X − F and A is g-closed, it follows that Cl(A) ⊆ X − F and thus F ⊆ X − Cl(A). This implies that F ⊆ (X − Cl(A)) ∩ (Cl(A) − A) = φ and hence F = φ.

Corollary 3.13: Let A a IFNg-closed set. Then A is IFNC if and only if *IFN*Cl(A) – A is IFNCS.

Proof: Let A be IFNg-closed set. If A is IFNCS, then we have *IFN*Cl(A) − A = φ which is IFNCS. Conversely, let *IFN*Cl(A) − A be IFNCS. Then, by Theorem 3.12, *IFN*Cl(A) − A does not contain any non-empty IFNC subset and since *IFN*Cl(A) − A is IFNC subset of itself, then *IFN*Cl(A) − A = φ. This implies that A = *IFN*Cl(A) and so A is IFNCS.

4 A Real Time Application

In this section, we discuss a real time application of *IFNTS* on one or more universal sets to multi criterion decision making. The application is discussed using intuitionistic fuzzy nano upper approximation space. In the case of telecommunication industries, there are various factors like tariff plans, affordable premium, network coverage, quality of service and location of service centres, which influence the public people interest on the company. Hence, *IF* relation provides the better relation between the public and telecommunication industries.

Consider $V = \{a_{1,} a_2, a_3, a_4, a_5\}$, in which a_1 is tariff plans; a_2 is affordable premium; a_3 is network coverage; a_4 is quality of service; a_5 is location of service centres and decisions $U = \{b_{1,} b_2, b_3, b_4, b_5\}$, in which b_1 is excellent; b_2 is best; b_3 is good; b_4 is satisfactory; b_5 is least affordable. Investors from various financial status are invited to the survey. Therefore, *(U, V, IFU_R, IFL_R)* be an *IFAS*, where $U = \{b_{1,} b_2, b_3, b_4, b_5\}$ and $V = \{a_{1,} a_2, a_3, a_4, a_5\}$.

If 17% investors rate excellent and 9% rate not excellent; 30% rate best; 3% rate not best; 15% give good; 10% rate not good; 10% rate satisfactory; 18% give not satisfactory; 24% give least affordable and 20% give not affordable, then we have the vector can be obtained as (.17,.09; .3,.03; .15,.1; .10,.18; .24,.2)*t* , where t represents the transpose. the decisions based on the other criteria are obtained as follows: (.3,.2; .37,.25; .35,.15; .25,.13; .3,.5)*t*,

(.6,.1; .35,.3; .2,.3; .3,.4; .1,.2)*t*, (.1,.5; .3,.2; .15,.4; .36,.2; .4,.15)*t* and (.23,.1; .2,.1; .3,.4; .1,.3; .2,.4)*t* . Based on the decision vectors, the *IF* relation from U to *V* is given by the following matrix.

$$R_{IF} = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{array} \begin{array}{c} \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \end{array} \\ \begin{bmatrix} .17, .09 & .3, .2 & .6, .1 & .1, .5 & .23, .1 \\ .3, .03 & .37, .25 & .35, .3 & .3, .2 & .2, .1 \\ .15, .1 & .35, .15 & .2, .3 & .15, .4 & .3, .4 \\ .10, .18 & .25, .13 & .3, .4 & .36, .2 & .1, .3 \\ .24, .2 & .3, .5 & .1, .2 & .4, .15 & .2, .4 \end{bmatrix} \end{array}$$

Two category of investors are considered, where right weightage for each criterion in *V* are V_1 = *(<b_1,.1,.20>,<b_2,.6,.13>,<b_3,.20,.30>,<b_4,.15,0.5>,<b_5,.1,.25>)* and V_2 = *(<b_1,.4,.3>, <b_2,.05,.15>, <b_3,0.4,0.42>,<b_4,0.13,0.32>,<u_5,0.12,0.31>)* respectively. Thus, by using *IF* upper approximation we have:

> *$IFU_R(V_1)$ = (<b_1,0.3,0.2>, <b_2,0.37,0.2>, <b_3,0.35,0.15>, <b_4,0.25,0.13>, <b_5,0.3,0.20>)t* and *$IFU_R(V_2)$ = (<b_1,0.4,0.2>, <b_2,0.35,0.25>, <b_3,0.2,0.15>, <b_4,0.3,0.15>, <b_5,0.24,0.3>)*t respectively.

From above, according to the principle of maximum membership, the decision for the first category of investors is best whereas for the second category is excellent.

5 Conclusion

We introduced intuitionistic fuzzy nano generalized closed sets in intuitionistic fuzzy nano topological spaces and an application of intuitionistic fuzzy nano topology in Multi Criterion Decision Making.

Competing Interests

Authors have declared that no competing interests exist.

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