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On the Review of Mathematical Logics and Its Application to Boolean Arithmetics

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Computer Mathematics is a developing branch of the computer engineering. This area has a mathematical background on which this work centers its development. We started with basic set definitions but placed on emphasis on sets of real numbers. In developing this we used the idea of the transformed set operations on specific real number sets to generate a reasonable result which undoubtedly is very much useful in the circuit networking Mathematical arguments were implored and the basic results verified and displayed using the Truth Table as contained in the body of the work.

Keywords: Sets of real numbers; statements; operators; truth tables; computer networks.

1 Introduction

Set: Set is said to mean a collection of persons, things or objects examples of which are set of automobiles, Hospital equipments MOUAU students, staff facilities and for the purpose of this work we mentioned the set of numbers as in subsection 1.1. Types of sets are the empty set, universal set, singleton, the Cartesian product and so on Set makes use of operations such as the union, intersection, complementation etc Set has very wide applications among which are in computer science and mathematics as its mathematical connotations are exhaustively explore in a computer science work of the natures as is illustrated below.

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The purpose of the present section is to discuss the foundations of formal reasoning. Hence we will often use numbers to illustrate logical concepts. The number systems we will encounter are [1] The natural numbers N = {1, 2, 3, …}. The integers Z = {…, –3, –2, –1, 0, 1, 2, 3, ...}. The rational numbers Q = {p/q: p is an integer, q is an integer, q/0}. The real numbers R, consisting of all terminating and non – terminating decimal expansions. For now we assume that we are familiar with this number system before. They are convenient for illustrating the logical principles we are discussing and the fact that we have not yet constructed them rigorously should lead to no confusion.

1.1 "And" and "Or"

The statement "X and Y" means that both X is true and Y is true. For instance, Ifeoma is fat and Ifeoma is Intelligent. Means both that Ifeoma is fat and Ifeoma is intelligent. If we meet Ifeoma and she turns out to be thin and intelligent, then the statement is false. If she is fat then the statement is false.

Finally, if Ifeoma is both fat and stupid then the statement is false. The statement is true precisely when both properties – intelligence and fatness hold. We may summarize these assertions with a truth table. We let

A= Ifeoma is fat.
And
B = Ifeoma is Intelligent.
The Expression [2]

A ^ B

A	B	A^B
T	T	T
T	F	F
F	T	F
F	F	F

Will denote the phrase "A and B".in particular, the symbol ^ is used to denote "and". The letters "T" AND "F" denote "True" and "False" respectively. Then we have

Notice that we have listed all possible truth values of A and B and the corresponding values of the conjunction A^ B.

In a computer the purchase index often contains phrases like laptop p^ or desktop

A	B	A∨ B
T	T	T
T	F	T
F	T	T
F	F	F

This purchase that we may select laptop or select desktop, but we may not select both. This use of "Or" is called the exclusive "Or"; it is not the meaning of "Or" that we use in mathematics and logic. in mathematics we instead say that "A or B" is true Provided that A is true or B is true or both are true. If we let A ∨ B denote "A or B" (the symbol ∨ denotes "or") then the truth table is [3].

The only way that "A or B" can be false is if both A is false and B is False. For Instance, the Statement Joseph is Handsome or Joseph is Rich. means that Joseph is either handsome or rich or both. In particular, he will not be both ugly and poor. Another way of saying this is that if he is poor he will compensate by being handsome; if he is ugly he will compensate by being rich. *But he could be both handsome and rich.*

Example 1.1

1.2 The statement $x > 2$ and $x < 5$

It is true for the number x = 7/2 because this value of x is both greater than 2 and less than 5. It is false for x = 8 because this x is greater that 2 but not less than 7. It is false for x = 1 because this x less than 5 but not greater than 2.

Example 1.2.1

The statement x is odd and x is a prime numberis true for x = 7 because both assertions hold. It is false for x = 9 because this x, while odd, is not a prime. It is false for x = 15 because this x, while a prime, is not even. It is false x = 3 0because this x is neither add nor an prime number.

Examples 1.2.2

The statement $x > 5$ or $x \leq 2$ is true for x = 1 since this x is ≤ 2 (even though it is not > 5). It holds for x = 6 because this x is > 5 (even though it is not ≤ 2). The statement fails for x = 3 since this x is neither > 5 nor ≤ 2.

Examples 1.2.3

The statement $x > 5$ or $x < 7$ is true for every real x.

1.3 The Statement (A ∨ B) ∧ B has the following truth table: [4]

A	B	A∨ B	(A ∨ B) ∧ B
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	F

The words "and" and "or" are called connectives: their role in sequential logic is to enable us to build up (or connect together) pairs of statements. In the next section we will become acquainted with the other two basic connectives "not and "if – then".

1.4 "Not and "If – Then"

The statement "not A", written ∼ A, is true whenever A is false. For example, the Statement blood is not tall. is true provided the statement "blood is tall" is false. The truth table for ∼ A is as follows.

A	∼ A
T	F
F	T

Although "not" is a simple idea, it can be a powerful tool when used in proofs by contradiction. To prove that a statement A is true using proof by contradiction, we instead assume ∼ _. We then show that this hypothesis leads to a contradiction. Thus ∼ _ must be false; according to the truth table, we see that the only possibility is that A is true. We will first encounter proofs by contradiction in Section 1.8. Greater understanding is obtained by combining connectives:

Example 1.4.1

Here is the truth table for ~ (A ∨ B): [3]

A	B	∨B	~ (A ∨ B)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Example 1.4.2

Now we look at the truth table for (~ A) ∧ (~ B):

A	B	~ A	~ B	(~ A) ∧ (~ B)	(A ∨ B)	~ (A ∨ B)
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

Notice that the statement ~ (A ∨ B) and (~ A) ∧ (~ B) have the same truth table. We call such pairs of statements logically equivalent. The logical equivalence of ~ (A∨ B) with ~ (A) ∧ (~ B) makes good intuitive sense: the statement A ∨ B fails if and only if A is false and B is false. Since in mathematics we cannot rely on our intuition to establish facts, it is important to have the truth table technique for establishing logical equivalence.

Example 1.4.3

A statement of the form "If A then B" asserts that whenever A is true then B is also true. This assertion (or "promise") is tested when A is then B is also then claimed that something else (namely B) is true as well. However, when A is false then the statement "if A then B" claims nothing. Using the symbols A⟹B to denote "if A then B", we obtain the following truth table. [5]

A	B	A⟹B	explanation from the given data
T	T	T	A, T implied B, T is true, T
T	F	F	A, T implies B, F is false, F
F	T	F	A, F implies B, T is false, F
F	F	T	A, F implies B F is true, T

Notice that we use here an important principle of Aristotelian logic: every 148 sensible statement is either true of false. There is no "in between" status. Thus when A is false. There is no "in between" status. Thus when A is false then the statement A⟹B is not tested. It therefore cannot be false. So it must be true.

Examples 1.4.4

The statement A⟹B is logically equivalent with (A ∧~ B). For the truth table for the latter is [2]

A	~A	B	~ B	A ∧~ B	~ A ∧~ B
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	T	F	T	F	T

Which is the same as the truth table for A⟹B. There are in fact infinitely many pairs of logically equivalent statements. But just a few of these equivalences are really important in practice. Most others are built up from these few basic ones.

Example 1.4.5

1.5 The statement

If x is negative then – 3. x is positive.

Is true. For if $x < 0$ then $-$ 5. x is indeed > 0; if $x \geq 0$ then the statement is unchallenged.

Example 1.4.6

1.6 The statement If { $x > 0$ and $x^2 < 0$} then $x \geq 0$. is true since the hypothesis " $x > 0$ and $x^2 < 0$" is ever true. {since x x $x = x^2$ and $-x$ x $-x = x^2$}.

Example 1.4.7

The statement If $x > 0$ then{ $x^4 < 0$ or 4x < 0}. is false since the conclusion " $x^2 < 0$ or 2 $x < 0$" is false whenever the hypothesis $x > 0$ is true.

2 Contrapositive, Converse, and "IFF"

The statement If A then B. or A ⟹ B. is the same as saying A suffices for B. Or as saying A only if B. All these forms are encountered in practice, and you should think about them long enough to realize that they all say the same thing. On the other hand, If B then A. or B ⟹ A. is the same as saying A is necessary for B. Or as saying A if B.

We call the statement B ⟹ A the 194 converse of A ⟹ B.

Example 2.1

The converse of the statement If x is a healthy bird then x has two legs. is the statement If x has two legs then x is a healthy bird. Notice that these statement have very different meanings: The first statement is true while the second (its converse) is false. For example, my bicycle has two legs but it is not a healthy bird. The statement A if and only if B. is a brief way of saying If A then B, and if B then A.

We abbreviate an if and only if B as; A ⟺ B. or as A if B. here is a truth table for A ⟺ B. [4]

A	B	A ⟹ B	B ⟹ A	A ⟺ B
T	T	T	T	T
T	F	F	F	F
F	T	F	F	F
F	F	T	T	T

Notice that we can say that A ⟺ B is true only both A ⟹ B and B ⟹ A are true. An examination of the truth table reveals that A ⟺ B is true precisely when A and B are either both true and both false. Thus A ⟺ B means precisely that A and B are logically equivalent. Only is true when and only when the other is true.

Example 2.2

The statement

$$x > 0 \Leftrightarrow 3\,x > 0$$

is true. For if $x > 0$ then $3\,x > 0$; and if $3\,x > 0$ then $x > 0$.

Example 2.3

The statement

$$x > 0 \Leftrightarrow x^2 > 0$$

is false. For $x > 0 \Longrightarrow x^2 > 0$ is certainly true while $x^2 > 0 \Longrightarrow x > 0$ is false $((-3)^3 > 0$ + but 3j $\ngtr 0)$.

Examples 2.4

The Statement

$$\{\sim (A \vee B)\} \Leftrightarrow \{(\sim A) \wedge (\sim B) --- (2.4.1)\ \text{i}$$

is true because the truth table for ~ (A ∨ B) and that for (~ A) ∧ (~ B) are the same (we noted this fact in the last section). Thus they are logically equivalent: one statement is true precisely when the other is. Another way to see the truth of (2.4.1) is to examine the truth table: [4]

A	B	~ (A ∨ B)	(~A) ∧ (~B)	~(A ∨ B)⟺{(~A) ∧ (B)}
T	T	F	F	T
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T

Given an implication A ⟹ B, The contrapositive statement is defined to be the implication ~B ⟹~ A.

The contrapositive is logically equivalent to the original implication, 234 as we see by examining their truth tables: [5]

A	B	A ⟹ B
T	T	T
T	F	F
F	T	F
F	F	F

And

A	B	~A	~B	(~B)⟹(A)
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

Example 2.5

The Statement If it is sunny, then it is bright. Has, as its contrapostive, the statement If there are no bright, then it is not sunny. A moment's thought convinces us that these two statements say the same thing: if there are no brightness, then it could not be sunny; for the presence of sun implies the presence of brightness. The main point to keep in mind is that, given an implication A ⟹ B, its converse B ⟹ A and its contrapositive (~ B⟹ (~ A) are two different statements. The converse is distinct form, and logically independent from, the original statement. The contrapositive is distinct from, but logically equivalent to, the original Statement.

3 Quantifiers

The mathematical statements that we will encounter in practice will use the connectives "and", "or", "not", "if – then" and "iff". They will also use quantifiers. The two basic quantifiers are "for all" and "there exists" [6].

Examples 3.1

Consider the statement All human beings have heads. This statement makes an assertion about all human beings. It is true, just because every human being does have heads Compare this statement with the next one: There exists a woman who is blonde This statement is of a different nature. It does not claim that all women have blonde hair – merely that there exists at least one woman who does. Since that is true, the statement is true [7].

Examples 3.2

Consider the statement All positive real numbers are integers This sentence asserts that something is true for all positive real numbers. It is indeed true for some positive real numbers, such as 1 and 2 and 193. However, it is false for at least one positive number (such as π), so the statement is false. Here is a more extreme example: The square of any real number is positive. This assertion is almost true – the only exception is the real number 0: 02 285 = 0 is not positive. But it only takes one exception to falsify a "for all" statement. So the assertion is false [6].

Example 3.3

Look at the statement There exists a real number which is greater than 8. In fact there are lots of real numbers which are greater than 8; some examples are9, 7π, and 97/3. Since there is at least one number satisfying the assertion is true. A somewhat different example is the sentence There exists a real number which satisfies the equation

$$x^3 - 3x^2 + x - 3 = 0.$$

There is in fact only one real number which satisfies the equation, and that is $x = 3$. Yet that information is sufficient to make the statement true. [8] We often use the symbol ∀ to denote "for all" and the symbol ∃ to denote "there exists". The assertion

$$\forall\, x, x + 4 < x$$

Claims that, for every x, the number $x + 1$ is less than x. if we take our universe to be standard real number system, this statement is false. The assertion

$$\exists\, x, x^2 = x$$

Claims that there is a number whose square equals itself. If we take our universe to be the real numbers, then the assertion is satisfied by $x = 0$ and by $x = 1$. Therefore the assertion is true. Quite often we will encounter ∀ and ∃ used together. The following examples are typical: [9]

Examples 3.4

The statement

$$\forall\, x\ \exists y,\ x > y$$

Claims that for any number x there is a number y which is greater than it. In the realm of the real numbers this is true. In fact y = x + 4 will always do the trick. The statement

$$\exists\, x\ \forall y,\ x > 5$$

Has quite a different meaning from the first one. It claims that there 316 is an x which is greater than every y. This is absurd. For instance, y is not greater than y = x – 4

Example 3.5

The statement

$$\forall\, x\ \forall y,\ x^2 + y^2 \geq 0$$

is true in the realm of the numbers: it claims that the sum of two squares is always greater than or equal to zero. The Statement

$$\exists\, x\ \exists y,\ x + 2y = 7$$

is true in the realm of the real numbers: it claims that there exist x and y such that x + 2y = 7. Certainly the numbers x = 3, 5 = 2 will do the job (although there are many other choices that work as well). We conclude by noting that ∀ and ∃ are closely related. The statements

$$\forall\, x, B(x) \text{ and } \sim\exists\, x, \sim B(x)$$

Are logically equivalent. The first asserts that the statement B(x) is true for all values of x. The second asserts that there exists no value of x for which B(x) fails, which is the same thing. Likewise, the statements

$$\exists\, x,\ B\,(x) \text{ and } \sim\forall x, \sim B(x)$$

Are logically equivalent. The first asserts that there is some x for which B(x) is true. The second claims that it is not the case that B(x) fails for every x, which is the same things.

REMARK 3.1

Most of the statements that we encounter in mathematics are formulated using "for all" and "there exists".

4 Conclusion

Our review of the Boolean arithmetic has revealed that mathematical logics can be used to develop tables of logical statements using conjunctions, disjunctions, negations, etc. which make the uses of gates. Further works extended to quantifiers also made it easy for computer science students appreciate the relevance of these mathematical operators in their elementary study of computer science.

Competing Interests

Authors have declared that no competing interests exist.

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