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Model Development of Reorder Point of an Inventory in an Imperfect Manufacturing System

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

This paper examines the concept of reorder point model of an inventory in an imperfect manufacturing system. The essence of the study is to know the point at which manufacturing firms should place an order for their raw materials. A model is developed which addresses the availability of raw materials in cases of variable demands and variable production lead times. The numerical data of innoson and bodex firm were obtained and used in testing the validity and robustness of the model. The result also showed the quantity to reorder provided the service level of the firm is known. This raw materials are made available via this model developed before manufacturing processes starts so as to avoid zero or excess stock. During production, defective items are produced with finished goods and as such rework process is necessary to convert those defectives into finished goods or products. Finally, this research is recommended for manufacturing industries in Nigeria for optimum performance.

Keywords: Reorder point; inventory; safety stock; production lead times; demand; confidence region; quantity etc.

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Track: 1 inventory modeling

Track: 2 manufacturing processes

1. INTRODUCTION

In the past one decade or so, managers of engineering projects world-wide are increasing coming under market pressure to reform, efficiently, and effectively maximize production through availability of raw materials as it is being done by their counterparts in other sector [1-3].

In Nigeria, most engineering firms have learnt through the hard way that only those with efficient and effective systems and techniques for provision of raw materials, production and inventory control can survive in the sector [4]. Hence, many firms are faced with problem of when to re-order for raw materials in a situation of variability in quantity demanded and production lead time [5-7].

In most manufacturing factories in Nigeria, there has been an incessant problem of when to re-order for raw materials to avoid the risk of stock out or excess inventory especially when there is no fixed quantity to be demanded at a fixed production lead time or when quantity demanded varies with fixed production lead time or vice versa [2,8]. Hence, this research provides an answer via a model on the timely availability of raw materials for production in a case of variable demand at a variable production lead time [9]. There has been existing models on re-order point but most of the models are developed on the basis of ideal situation when companies have fixed quantity to be demanded with a fixed production lead time or a situation where either demand or lead time varies and the other becomes constant [8-11]. Considering some external environmental factors in Nigeria, this approach is seem to be naive in our manufacturing industries. Hence, the development of re-order point model with variable quantity demanded and variable production lead time which takes into account all the natural and assignable variation unlike the previous or already existing model which fails to address variable condition such as change in customers demand which affects the overall demand of the company's raw materials and poor road network which affects the production lead time [12].

Furthermore, this model differentiated itself from the existing models of re-order point by its total approach of integrating all the factors that hinders timely deliverance of raw materials as well as customers perspective at a particular instant [3,5,13-15]. It controls in totality the inventory level of production (Raw materials). However, this approach does not exist without the following components such as normal distribution with zero (0) mean and variance one (1), test statistics (z), safety stock, confidence region, upper limits, lower limits, level of significance, number of standard deviations etc [6,16-18].

Finally, the result of the calculation showed the robustness of the model though with a specific limitation that the service level of each firm wishing to apply and the model must be known.

2. MODEL NOTATIONS

The following notations were used in the model formulation below;

$\bar{d}$ = Average demand (weeks)

$\bar{L}$ = Average Lead time (days)

Z_α = Number of standard deviations

α = Level of significance

σ_d^2 = Variance of the demand

σ_L^2 = Variance of the Lead time

$S_{\bar{L}\bar{d}}$ = Standard errors of the mean of the led time and mean of the demand.

$\sim$N (0, 1) = Follows normal distribution with zero mean and variance 1

Z = test statistic

P = Probability

S_s = Safety stock

(1 – α) = Confidence region

(1 – α) 100% = Shows that we are (1-α) 100% confidence that the safety stock will lie in the interval.

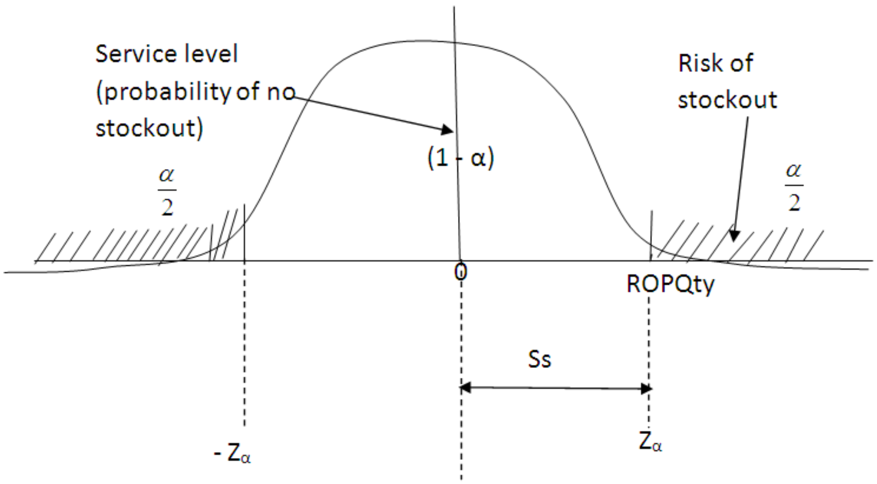
L = Lower Limits

U = Upper Limits

ROP = Re-order point

3. MODEL FORMULATION

The Re-order point (ROP) based on a normal distribution of variable lead time and variable demand may be represented in the diagram below;



To establish the safety stock region, we employ the following test statistic (Z)

$$\text{ie } Z = \frac{Sample \quad mean - Population \quad mean}{Standard \quad error \quad of \quad the \quad mean}$$

$$\Rightarrow Z = \frac{\overline{L}\overline{d} - E(\overline{L}).E(\overline{d})}{S_{\overline{L}\overline{d}}}$$

$$\because Z = \frac{\overline{d}x\overline{L} - Ss}{\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}} \quad N(0,1) \quad (1)$$ Satorra and Bentiler(2001).

where $S_{\overline{L}\overline{d}} = \sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}$

The confidence interval for the safety stock (Ss) is given by

$$P[-Z_\alpha \leq Z \leq Z_\alpha] = (1-\alpha) \text{ 100\%} \quad (2)$$

Substituting for Z in equation (2) above, we have;

$$P\left[-Z_\alpha \leq \frac{\overline{d}x\overline{L} - Ss}{\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}} \leq Z_\alpha\right] = (1-\alpha) \text{ 100\%} \quad (3)$$

Multiply each term in the bracket by

$\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}$ *we have*

$$P\left[-Z_\alpha\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}\right] \leq \sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2} \quad X \quad \frac{\overline{d}\,\overline{L} - Ss}{\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}} \leq Z_\alpha\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}\Bigg] = \quad (1-\alpha)100\%$$

$$\Rightarrow P\left[-Z_\alpha\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2} \leq \overline{d}\overline{L} - Ss \leq Z_\alpha\sqrt{\overline{L}\sigma_d^2 + \overline{d}^2\sigma_L^2}\right] = (1-\alpha)100\% \quad (4)$$

Subtract $\bar{d}\bar{L}$ from each term in the bracket, we have that;

$$P\left[-\bar{d}\bar{L} - Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2} \le \bar{d}\bar{L} - Ss - \bar{d}\bar{L} \le -\bar{d}\bar{L} + Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L}\right] = (1-\alpha)100\% \ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$\Rightarrow P\left[-\bar{d}\bar{L} - Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2} \le -S_s \le -\bar{d}\bar{L} + Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}\right] = (1-\alpha)100\% \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Now multiply each term in the bracket by – 1 to make S_s positive.

$$ie \qquad P\left[\bar{d}\bar{L} + Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2} \ge S_s \ge \bar{d}\bar{L} - Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}\right] = (1-\alpha)100\% \ldots\ldots\ldots\ldots\ldots \quad (7)$$

Using the fact that if a ≥ x ≥ b ≡ b ≤ x ≤ a, we can write that

$$P\left[\bar{d}\bar{L} - Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2} \le \quad S_s \le \bar{d}\bar{L} \pm Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}\right] = (1-\alpha)100\,\% \ldots\ldots\ldots \ldots\ldots \quad (8)$$

Let L and U denote Lower and Upper Limits respectively.

$$\Rightarrow L = \bar{d}\bar{L} - Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(9). \qquad And$$

$$U = \bar{d}\bar{L} + Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Since stock cannot be negative, equation (10) becomes useful. So, we use the upper limit to find our Re-order point (ROP)
ie

$$\text{ROP} = \bar{d}\bar{L} + Z_\alpha\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}$$

4. ASSUMPTIONS OF THE MODEL

(i) One of the assumptions is that the test statistic Z, is normally distributed with 0 mean and variance 1
(ii) The Interval of the safety stock is taken to be non-negative
(iii) The model assumes variable demands and variable lead time
(iv) Sample values were used to estimate the population values. Eg sample means $\bar{d}$, $\bar{L}$ were used to estimate the population means of d and L and sample variances σ_L^2 and σ_d^2 were used to estimate their population variances.
(v) The model is valid if the service level of the flow station is known.

5. RESULTS SPECIFICATION

The results of the research/model formulated will be applied to manufacturing industries wishing to adopt Just-in-time through Reorder point model of inventory control.

The model will also be suitable in solving problems of variability in both the quantity demanded and as well the production lead times which is aimed at knowing the proper time a firm should Re-order or their quantities to be always available for Work. This is also geared towards having a good inventory level of their firm. It ensures Just-in-time of their products there by avoiding risk of stock out or excess stock, to avoid waste of any kind. This also minimizes the cost of production.

6. TESTING OF MODEL FORMULATION

This model as formulated to know the reorder point when there are variable demand and lead time of activities to ensure Just – in – time of their products which prevents stock out or excess stock of their inventory level.

The innoson factory data was obtained having known there service level was used in testing the validity, usefulness and robustness of the model.

7. INNOSON FACTORY DATA WITH SERVICE LEVEL OF 96% WITH VARIABLE DEMANDS AND LEAD TIMES

Table 1. Innoson factory data with variable demands and lead times

Weeks	1	2	3	4	5	6	7	Mean($\bar{x}$)	Variance(σ^2)	Standard deviation
Demands (WKS)	50	60	45	80	75	65	55	$(\bar{d})$ = 61	σ_d^2 = 141	$\sqrt{\sigma_d^2} = 11.87$
Lead times (DAYS)	5	6	4	7	6	5	4	$(\bar{L}) = 5$	σ_L^2 = 1.14	$\sqrt{\sigma_i^2} = 1.07$

8. DETAILS OF THE CALCULATION

Mean of demand $(\bar{d}) = \dfrac{\sum d}{n} = \dfrac{50+60+45+80+75+65+55}{7}$

$$\therefore (\bar{d}) = 61$$

Mean of lead time $(\bar{L}) = \dfrac{\sum L}{n} = \dfrac{5+6+4+7+6+5+4}{7} = 5$

Variance of Demand (σ_d^2) = $\dfrac{(d_1-\bar{d})^2+(d_2-\bar{d})^2+(d_3-\bar{d})^2+...+(d_n-\bar{d})^2}{n}$

$$\therefore \sigma_d^2 = \frac{(50-61)^2+(60-61)^2+(45-61)^2+(80-61)^2+(75-61)^2+(65-61)^2+(55-61)^2}{7}$$

$$\sigma_d^2 = 987/7 = 141$$

Variance of lead time (σ_L^2) = $\dfrac{(L_1-\bar{L})^2+(L_2-\bar{L})^2+(L_3-\bar{L})^2+....+(L_n-\bar{L})^2}{n}$

$$\sigma_L^2 = \frac{(5-5)^2+(6-5)^2+(4-5)^2(7-5)^2+(6-5)^2+(5-5)^2+(4-5)^2}{7}$$

$$\therefore \sigma_L^2 = \frac{8}{7} = 1.14$$

Standard Deviation of demand $\left(\sqrt{\sigma_d^2}\right) = \sqrt{141} = 11.87$

Standard Deviation of lead time $\left(\sqrt{\sigma_L^2}\right) = \sqrt{1.41} = 1.07$

At service level of 96%=0.96, Z_α=1.75 (from the table of normal distribution Service level).

Applying the R O P Model, we have;

$$ROP = \bar{d}\bar{L} + Z_\alpha \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2 \sigma_L^2}$$

$$ROP = 61 \times 5 + 1.75 \sqrt{5x141 + 61^2 x1.14}$$

$$ROP = 305 + 1.75 \sqrt{705 + 4241.94}$$

$$ROP = 305 + 1.75 \sqrt{4946.94}$$

R O P = 305 +1.75 x 70.33 = 305 +123

∴ R O P = 428 quantities

Hence, the factory places an order at an inventory of 428 quantities of pipes and other equipment (raw materials) to allow Just-In-time of their products before a stock out is noticed.

9. BODEX NUT MANUFACTURING COMPANY WITH SERVICE LEVEL OF 93% WITH VARIABLE QUANTITIES OF DEMANDS AND PRODUCTION LEAD TIMES OF THEIR PRODUCTS

Table 2. Mean, variance and standard deviation of demands and lead time production

Weeks	1	2	3	4	5	6	7	Mean ($\bar{X}$)	Variance (σ^2)	Standard Deviation
Demands (Weeks)	30	20	50	45	60	70	65	($\bar{d}$) =4 9	(σ_d^2) = 291	$\sqrt{\sigma_d^2} = 17$
Lead times (Days)	4	3	4	5	6	7	5	($\bar{L}$) = 4.9	(σ_L^2) = 1.6	$\sqrt{\sigma_L^2} = 1.3$

Mean of demand ($\bar{d}$) = $\frac{\sum d}{n} = \frac{30+20+50+45+60+70+65}{7} = 49$

Mean of lead-time ($\bar{L}$) = $\frac{\sum L}{n} = \frac{4+3+4+5+6+7+5}{7} = 4.9$

Variance of demand (σ_d^2) = $\frac{(d_1-\bar{d})^2 + (d_2-\bar{x})^2 + (d_3-\bar{d})^2 + ... + (d_n-\bar{d})^2}{n}$

σ_d^2 =

$$\frac{(30-49)^2 + (20-49)^2 + (50-49)^2 + (50-49)^2 + (45-49)^2 + (60-49)^2 + (70-49)^2 + (65-49)^2}{7}$$

$$\sigma_d^2 = \frac{2037}{7} = 291$$

Variance of lead time (σ_L^2) = $\frac{(L_1 - \overline{L})^2 + (L_2 - \overline{L})^2 + ... + (L_n - \overline{L})^2}{n}$

$$\sigma_L^2 = \frac{(4-4.9)^2 + (3-4.9)^2 + (4-4.9)^2 + (5-4.9)^2 + (6-4.9)^2 + (7-4.9)^2 + (5-4.9)^2}{7}$$

$$\sigma_L^2 = \frac{10.87}{7} = 1.6$$

Standard Deviation of demand $\left(\sqrt{\sigma_d^2}\right) = \sqrt{291} = 17$

Standard Deviation of lead time $\left(\sqrt{\sigma_L^2}\right) = \sqrt{1.6} = 1.3$

At service level of 93% = 0.93, Z_α = 1.48 (from the table of normal distribution service level).

Applying the Developed R O P model, we have that;

$$\text{R O P} = \overline{d}\,\overline{L} + Z_\alpha \sqrt{\overline{L}\sigma_d^2 + \overline{d}^2 \sigma^2 L}$$

$$\text{R O P} = 49 \times 4.9 + 1.48 \sqrt{4.9x291 + 49^2 x1.6}$$

$$\text{R O P} = 240.1 + 1.48 \sqrt{1425.9 + 3841.6}$$

$$\text{R O P} = 240.1 + 1.48 \sqrt{5267.5} = 240.1 + 1.48x72.6$$

R O P = 240.1+107.45 = 347.55

$\therefore$ R O P $\cong$ 348 quantities

Thus, the company reorders at an inventory of 348 quantities of their materials to ensure J I T of their products before a zero stock is experienced.

10. CONCLUSION

This study revealed a positive, useful model of reorder point of variable production lead time as well as variable demand which addressed the timely availability of raw materials before production processes. The validity of the model is attained only when the service level of such firm is known and this avoids zero or excess stock of an inventory. It ensures or serves as a measure of buffer raw materials in single stage/multi stage production process where rework and scrap production/detection are possible. Hence, the model will make significant and meaningful contribution in solving the problem of unavailability and inefficiency in our production systems especially in cases of variable lead time and demand in an imperfect manufacturing systems. It would be therefore recommended for all production industries in Nigeria to ensure efficiency and effectiveness of operation in the face of increasing market/ customers' demands for competitive advantage in our global economy.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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