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# **Method for Constraining Light Speed Anisotropy by Using Fiber Optics Gyroscope Experiments**

## **A. Sfarti1\***

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#### *Author's contribution*

*This work was carried by the author. Author AS designed the study, performed the experiment and the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. The author read and approved the final manuscript.*

*Research Article*

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### **ABSTRACT**

The Mansouri-Sexl theory is a well known test theory of relativity. Mansouri and Sexl dealt with the theory of the Michelson-Morley, Kennedy-Thorndike and Ives-Stilwell experiments but left out the very interesting Sagnac experiment. In the following paper we will present a

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<sup>1</sup> *LOC Berkeley, CS Dept, 387 Soda Hall, Berkeley, CA 94720, USA.<br> Author's contribution<br> Prime way of detection of the author. Author AS designed the study, performed the experiment and the straintin* experiment using fiber optic gyroscopes (FOG) where  $L$  is the length of the fiber and  $\omega$ is the angular speed of the FOG. We show how the fiber optics gyroscopes are used for constraining light speed anisotropy in the framework of the Mansouri-Sexl test theory. We also show an interesting amplification effect due to the use of the Mansouri-Sexl slow clock transport equations in conjunction with FOGs. Our paper is divided into four main sections: in the first one we give an overview of the Mansouri-Sexl test theory of special relativity, in the second one we give a historical perspective of the Sagnac experiment, in the third section we formulate the Mansouri-Sexl theory for the Sagnac experiment and we conclude with experimental setup and results.

*\_*

*Keywords: Mansouri-sexl test theory; light speed anisotropy; fiber optic gyroscopes.*

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#### **1. INTRODUCTION - THE MANSOURI - SEXL TEST THEORY**

The test theories [1-4] of special relativity are used to examine potential alternate theories to special relativity (SR) - such alternate theories predict particular values of the parameters of the test theory, which may easily be compared to values determined by experiments. The existing experiments put rather strong constraints on any alternative theory. One of these theories, the Robertson-Mansouri-Sexl theory, starts by admitting that there is one preferential inertial frame  $\Sigma$  in which the light propagates isotropically. In such a frame, Physical Review & Research International, 3(3): 161-175, 2013<br>
1. **INTRODUCTION - THE MANSOURI - SEXL TEST THEORY**<br>
The test theories [1-4] of special relativity are used to examine potential alternate theories to<br>
specia is the refraction index of the optic fiber. All other frames in motion with respect to  $\Sigma$  are considered non-preferential and the light speed is anisotropic. The light speed in the non preferential frames can be deduced via simple calculations described in<sup>3</sup>. We start with the Mansouri-Sexl transforms (with c=1): Physical Review & Research International, 3(3): 161-175, 2013<br>
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theories [1-4] of special relativity are used to examine potential alternate theorie n a refractive medium is  $c_0 = c/n$  where c is the light speed in vacuum and n<br>tion index of the optic fiber. All other frames in motion with respect to  $\Sigma$  are<br>non-preferential and the light speed is anisotropic. The ligh

$$
\mathbf{x} = d(v)\mathbf{X} + \frac{b(v) - d(v)}{v^2}\mathbf{v}(\mathbf{v}\mathbf{x}) - b(v)\mathbf{v}T
$$
  
\n
$$
t = a(v)T + \varepsilon(v)\mathbf{x}
$$
\n(1.1)

where **v** is the relative velocity between S and  $\Sigma$ ,  $(x,t)$  are the coordinates in S while  $(X,T)$ represent their correspondents in  $\Sigma$ . Exactly like in the original Mansouri-Sexl paper [4] by higher we obtain [2]: =  $d(v)\mathbf{X} + \frac{b(v) - d(v)}{v^2} \mathbf{v}(\mathbf{vx}) - b(v)\mathbf{v}T$ <br>
=  $a(v)T + \mathbf{\varepsilon}(v)\mathbf{x}$ <br>
the relative velocity between S and  $\Sigma$ , (x,t) are the coordination their correspondents in  $\Sigma$ . Exactly like in the original Mans<br>
ng the lig

$$
\frac{c_{\pm}(\theta)}{c_0} \approx 1 \mp \frac{v}{c_0} (1 + 2\alpha) \cos \theta \tag{1.2}
$$

where  $\theta$  is the angle between the light ray direction and the x axis. Expression (1.2) is an approximation valid if slow clock transport [2-4] synchronization has been used. In this case, the following expressions also hold [2]:

is the refraction index of the optic. All order frames in motion with respect to ∠ are  
considered non-preferential and the light speed is anisotropic. The light speed in the non-  
preferential frames can be deduced via simple calculations described in<sup>3</sup>. We start with the  
Mansouri-Sexl transforms (with t=1):  

$$
x = d(v)X + \frac{b(v) - d(v)}{v^2}v(xx) - b(v)vT
$$
  

$$
t = a(v)T + ε(v)x
$$
  
where **v** is the relative velocity between S and  $\Sigma$ . (x,t) are the coordinates in S while (X,T)  
represent their corresponds in  $\Sigma$ . Exactly like in the original Mansouri-Sexl paper [4] by  
transforming the light cone  $X^2 - c_0^2T^2 = 0$  into S and by neglecting the terms in  $v^2$  and  
higher we obtain [2]:  

$$
\frac{c_+(θ)}{c_0} \approx 1 \mp \frac{v}{c_0} (1 + 2\alpha) \cos \theta
$$
 (1.2)  

$$
\frac{c_+(θ)}{c_0} \approx 1 \mp \frac{v}{c_0} (1 + 2\alpha) \cos \theta
$$
 (1.2)  

$$
a(v) \approx 1 + \alpha(v) \frac{v^2}{c^2}
$$

$$
b(v) \approx d(v) \approx 1
$$
 (1.3)  

$$
\varepsilon \approx 2\alpha v
$$
 According to Mansour and Sekl, the one-way light speed is a measurable quantity in this  
the light speed anisotropy. We will exploit this property in the Mansouri-Sexl theory of the  
FOG experiment constructed later in our paper.  
On the other hand, according to Mansouri and Sekl [2], if Einstein clock synchronization is  
used, no first order effects exist and the second order effects are expressed as:

According to Mansouri and Sexl, the one-way light speed is a measurable quantity in this the light speed anisotropy. We will exploit this property in the Mansouri-Sexl theory of the FOG experiment constructed later in our paper.

On the other hand, according to Mansouri and Sexl [2], if Einstein clock synchronization is used, no first order effects exist and the second order effects are expressed as:

*c*(
$$
\theta
$$
)  $\approx$  
$$
\frac{c_0}{1 + (\beta - \delta - 1/2)(\frac{v}{c_0})^2 \sin^2 \theta + (\alpha - \beta + 1)(\frac{v}{c_0})^2}
$$
 (1.4)  
where  $\beta$ ,  $\delta$  are parameters originating from the Taylor expansion of *b*, *d* respectively. In this case, the Sagnac effect cannot be used for measuring light speed anisotropy because the second order effects are too small to measure for any reasonable value for the speed *v*.  
1.1. THE SPECIAL RELATIVITY THEORY OF THE SAGNAC EXPERIMENT USING FOG

this case, the Sagnac effect cannot be used for measuring light speed anisotropy because the second order effects are too small to measure for any reasonable value for the speed *v* .

#### **1.1. THE SPECIAL RELATIVITY THEORY OF THE SAGNAC EXPERIMENT USING FOG**

A fiber optic gyroscope (FOG) senses changes in orientation, thus performing the function of a mechanical gyroscope. However its principle of operation is instead based on the interference of light which has passed through a coil of optical fiber. Two beams from a laser are injected into the same fiber but in opposite directions. Due to the Sagnac effect, the beam travelling against the rotation experiences a slightly shorter path delay than the other beam. The resulting differential phase shift is measured through interferometry, thus translating one component of the angular velocity into a shift of the interference pattern which is measured.



**Fig. 1. Explanation of the sagnac experiment**

The right hand side of Fig. 1 illustrates what happens if the loop itself is rotating. The symbol  $\alpha$  denotes the angular displacement of the loop during the time required for the pulses to travel once around the loop. For any positive value of  $\alpha$ , the pulse traveling in the same direction as the rotation of the loop must travel a slightly greater distance than the pulse traveling in the opposite direction. As a result, the counter-rotating pulse arrives at the "end" point slightly earlier than the co-rotating pulse. Quantitatively, if we let  $\omega$  denote the angular speed of the loop, then the circumferential tangent speed of the end point is  $\omega R$ . The respective angles traveled by the two light fronts are:

*Physical Review & Research International, 3(3): 161-175, 2013*\n
$$
\phi_{+} = 2\pi + \alpha_{+} = \frac{c_{+}l_{+}}{R}
$$
\n(2.1)

\nfor the co-rotating front

\n
$$
\phi_{-} = 2\pi - \alpha_{-} = \frac{c_{-}l_{-}}{R}
$$
\n(2.2)

\nfor the counter-rotating front,

\nwhere  $c_{+} = c_{-} = c$  in vacuum and:

\n
$$
\alpha_{+} = \omega t_{+}
$$
\n(2.3)

\nfor the co-rotating front

\n
$$
\alpha_{-} = \omega t_{-}
$$
\n(2.4)

for the co-rotating front

co-rotating front  
\n
$$
\phi_{-} = 2\pi - \alpha_{-} = \frac{c_{-}t_{-}}{R}
$$
\ncounter-rotating front,  
\n
$$
c_{+} = c_{-} = c
$$
 in vacuum and:  
\n
$$
\alpha_{+} = \omega t_{+}
$$
\n(2.3)  
\nco-rotating front  
\n
$$
\alpha_{-} = \omega t_{-}
$$
\n(2.4)  
\ncounter-rotating front.  
\n(2.3) in (2.1) and (2.4) in (2.2) we get:

for the counter-rotating front,

$$
\alpha_{+} = \omega t_{+} \tag{2.3}
$$

for the co-rotating front

$$
\alpha_{-} = \omega t_{-} \tag{2.4}
$$

for the counter-rotating front. Substituting (2.3) in (2.1) and (2.4) in (2.2) we get:

$$
t_{+} = \frac{2\pi R}{c - \omega R} \tag{2.5}
$$

for the co-rotating front

$$
\phi_{+} = 2\pi + \alpha_{+} = \frac{c_{+}l_{+}}{R}
$$
\n
$$
\text{co-rotating front}
$$
\n
$$
\phi_{-} = 2\pi - \alpha_{-} = \frac{c_{-}l_{-}}{R}
$$
\n
$$
\text{counter-rotating front}
$$
\n
$$
c_{+} = c_{-} = c \text{ in vacuum and:}
$$
\n
$$
\alpha_{+} = \omega t_{+}
$$
\n
$$
\alpha_{-} = \omega t_{-}
$$
\n
$$
\text{co-rotating front}
$$
\n
$$
\alpha_{-} = \omega t_{-}
$$
\n
$$
\text{counter-rotating front}
$$
\n
$$
t_{+} = \frac{2\pi R}{c - \omega R}
$$
\n
$$
\text{co-rotating front}
$$
\n
$$
t_{+} = \frac{2\pi R}{c + \omega R}
$$
\n
$$
\text{co-rotating front}
$$
\n
$$
t_{-} = \frac{2\pi R}{c + \omega R}
$$
\n
$$
\text{(2.6)}
$$
\n
$$
\text{counter-rotating front}
$$
\n
$$
t_{-} = \frac{2\pi R}{c + \omega R}
$$
\n
$$
\text{(2.7)}
$$

for the counter-rotating front. From (2.5) and (2.6) it follows that:

$$
\phi_{-} = 2\pi - \alpha_{-} = \frac{c_{-}t_{-}}{R}
$$
\ncounter-rotating front,

\n
$$
c_{+} = c_{-} = c \text{ in vacuum and:}
$$
\n
$$
\alpha_{+} = \omega t_{+}
$$
\nco-rotating front

\n
$$
\alpha_{-} = \omega t_{-}
$$
\ncounter-rotating front

\n
$$
\alpha_{-} = \frac{2\pi R}{c - \omega R}
$$
\n(2.4)

\ncounter-rotating front

\n
$$
t_{+} = \frac{2\pi R}{c - \omega R}
$$
\nco-rotating front

\n
$$
t_{-} = \frac{2\pi R}{c + \omega R}
$$
\ncounter-rotating front

\n
$$
t_{-} = \frac{2\pi R}{c + \omega R}
$$
\ncounter-rotating front. From (2.5) and (2.6) it follows that:

\n
$$
\Delta T_{total} = t_{+} - t_{-} = \frac{4\pi R^{2} \omega}{c^{2} - R^{2} \omega^{2}} = \frac{4A\omega}{c^{2} - R^{2} \omega^{2}}
$$
\n(2.7)

(2.5)<br>
(2.5)<br>
the (2.7) in (2.5) and (2.6) it follows that:<br>  $\frac{4\pi R^2 \omega}{r^2 - R^2 \omega^2} = \frac{4A\omega}{c^2 - R^2 \omega^2}$ <br>
interferometer loop. The above is the exact formula. For  $R\omega \ll c$ <br>
used in practice for detecting angular speed (2.3)<br>
(2.4)<br>
the divideo (2.4) in (2.2) we get:<br>
(2.5)<br>
(2.6)<br>
the From (2.5) and (2.6) it follows that:<br>
(2.6)<br>
the From (2.5) and (2.6) it follows that:<br>  $\frac{4\pi R^2 \omega}{R^2 \omega^2} = \frac{4A\omega}{c^2 - R^2 \omega^2}$  (2.7)<br>
interferomete bounder-louding front<br>  $\frac{1}{r} = 0.7$  *F*  $\frac{1}{r} = 0.7$  *C* is the axe of the control of the control of the axe of the interference of the condition of the axe of the interference for detecting angular speed via the Sagn (2.3)<br> **c** (2.4) in (2.2) we get:<br> **c** (2.4) in (2.2) we get:<br>
(2.5)<br> **c** (2.6)<br> **c** (2.6)<br> **c** (2.6)<br> **c** (2.6)<br> **c** (2.6)<br> **c** (2.6)<br> **c** (2.7)<br> **c** interferometer loop. The above is the exact formula. For  $Ro << c$ <br> **c** us counter-rotating front,<br>  $\alpha_z = c = c$  in vacuum and:<br>  $\alpha_z = e\pi_z$  (2.3)<br>
co-rotating front<br>  $\alpha_z = e\pi_z$  (2.4)<br>
counter-rotating front<br>  $\alpha_z = e\pi_z$  (2.4)<br>
counter-rotating front<br>  $t_z = \frac{2\pi R}{c - \omega R}$  (2.5)<br>
co-rotating front<br>  $t =$ or the co-rotating front<br>
for the co-rotating front<br>
for the counter-rotating front<br>
for the counterferometer loop. (2.4)<br>
for the corrotating front<br>  $t_x = \frac{2\pi R}{c + \omega R}$  (2.5)<br>
for the corrotating front<br>
for the counterfe we recover the formula used in practice for detecting angular speed via the Sagnac experiment [5,6]:

$$
\Delta T_{total} = \frac{4A\omega}{c^2} \tag{2.8}
$$

 $\alpha_z = \omega t$ <br>
counter-rotating front.<br>  $t_+ = \frac{2\pi R}{c - \omega R}$  (2.5) and (2.4) in (2.2) we get:<br>  $t_+ = \frac{2\pi R}{c - \omega R}$  (2.5)<br>
co-rotating front<br>  $\iota_z = \frac{2\pi R}{c + \omega R}$  (2.6)<br>
counter-rotating front. From (2.5) and (2.6) it follow The formula shows that the phase difference between the two counter-propagating light signals is, at low angular speeds, proportional to the angular speed and to the area enclosed by the interferometer loop. The first to perform a ring interferometer experiment aimed at observing the correlation of angular velocity and phase-shift was G. Sagnac [6] in 1913 with

the purpose of detecting "the effect of the relative motion of the ether". In 1926 a very ambitious ring interferometry experiment was set up by A. Michelson and H.Gale [7]. The aim was to find out whether the rotation of the Earth has an effect on the propagation of light in the vicinity of the Earth. The Michelson-Gale experiment used a very large ring interferometer, with a perimeter of 1.9 kilometer, so it was large enough to detect the angular velocity of the Earth. The outcome of the experiment was that the angular velocity of the Earth as measured by astronomical methods was confirmed to within measuring accuracy. The situation is a little more complicated in the case of using a fiber optic of refraction index *n* :

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\nby of the Earth. The outcome of the experiment was that the angular velocity of the  
\nas measured by astronomical methods was confirmed to within measuring accuracy;  
\n(ituation is a little more complicated in the case of using a fiber optic of refraction index  
\n
$$
c_{+} = \frac{\frac{c}{n} + \omega R}{1 + \frac{\omega R}{n c}}
$$
\n
$$
c_{-} = \frac{\frac{c}{n} - \omega R}{1 - \frac{\omega R}{n c}}
$$
\n
$$
t_{+} = \frac{2\pi R}{c_{+} - \omega R} = 2\pi R \frac{1 + \frac{\omega R}{n c}}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{2\pi R}{c_{+} - \omega R} = 2\pi R \frac{1 - \frac{\omega R}{n c}}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{2\pi R}{c_{+} - \omega R} = 2\pi R \frac{1 - \frac{\omega R}{n c}}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n
$$
t_{-} = \frac{4\pi R^{2} \omega}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\n

Substituting (2.9) into (2.5)-(2.6):

$$
c_{-} = \frac{\frac{c}{n} - \omega R}{1 - \frac{\omega R}{nc}}
$$
  
\nFitting (2.9) into (2.5)-(2.6):  
\n
$$
t_{+} = \frac{2\pi R}{c_{+} - \omega R} = 2\pi R \frac{1 + \frac{\omega R}{nc}}{\frac{c}{n} - \frac{(\omega R)^{2}}{nc}}
$$
 (2.10)  
\n
$$
t_{-} = \frac{2\pi R}{c_{+} + \omega R} = 2\pi R \frac{1 - \frac{\omega R}{nc}}{\frac{c}{n} - \frac{(\omega R)^{2}}{nc}}
$$
 (2.11)  
\n
$$
t_{-} = \frac{2\pi R}{c_{+} + \omega R} = 2\pi R \frac{1 - \frac{\omega R}{nc}}{\frac{c}{n} - \frac{(\omega R)^{2}}{nc}}
$$
 (2.12)  
\n
$$
\Delta T_{total\_SR} = t_{+} - t_{-} = \frac{4\pi R^{2} \omega}{c^{2} - R^{2} \omega^{2}}
$$
 (2.12)  
\n
$$
= \frac{\Delta T_{total\_SR}}{\Delta T_{total\_SR}} = t_{+} - t_{-} = \frac{4\pi R^{2} \omega}{c^{2} - R^{2} \omega^{2}}
$$
 (2.12)  
\n
$$
= \frac{\Delta T_{total\_SR}}{\Delta T_{total\_SR}} = \frac{\Delta T_{total}}{\Delta T_{total\_SR}} = \frac{\Delta T_{total}}{\Delta T_{total\_SR}} = \frac{(\Delta T_{total\_SR})}{(\Delta T_{total}/T_{ion})} = \frac{\Delta T_{total}}{\Delta T_{total\_SR}} = \frac{(\Delta T_{total}/T_{ion})}{(\Delta T_{total}/T_{ion})} = \frac{\Delta T_{total}}{\Delta T_{total\_SR}} = \frac{(\Delta T_{total}/T_{ion})}{(\Delta T_{total}/T_{ion})} = \frac{\Delta T_{total}}{\Delta T_{total\_SR}} = \frac{(\Delta T_{ion}/T_{ion})}{(\Delta T_{ion}/T_{ion})} = \frac{\Delta T_{ion}}{\Delta T_{ion}}
$$

for the co-rotating front

$$
\frac{1}{n} - \frac{2\pi R}{n c}
$$
\nthe co-rotating front

\n
$$
t_{-} = \frac{2\pi R}{c_{+} \omega R} = 2\pi R \frac{1 - \frac{\omega R}{n c}}{1 - \frac{(\omega R)^{2}}{n c}}
$$
\nthe counter-rotating front, resulting into a total time:

\n
$$
\Delta T_{total\_SR} = t_{+} - t_{-} = \frac{4\pi R^{2} \omega}{c^{2} - R^{2} \omega^{2}}
$$
\n(2.12)

\nessingly enough, the outcome of the experiment does not depend on the refraction index, i.e., the SR prediction from expression (2.12) fully coincides with the rimental results [9]. One of the important advantages of FOGs, besides the absence of

for the counter-rotating front, resulting into a total time:

$$
\Delta T_{total\_SR} = t_+ - t_- = \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2}
$$
\n(2.12)

 $rac{R}{(R - C)^2}$  (2.10)<br>  $rac{C}{(R - C)^2}$  (2.11)<br>  $rac{R}{(R - C)^2}$  (2.11)<br>  $rac{4\pi R^2 \omega}{(R - C)^2 \omega^2}$  (2.12)<br>
of the experiment does not depend on the refraction index<br>
elimportant advantages of FOGs, besides the absence of<br>
the opt Interestingly enough, the outcome of the experiment does not depend on the refraction index of the fiber optic. The SR prediction from expression (2.12) fully coincides with the experimental results [9]. One of the important advantages of FOGs, besides the absence of any moving parts is the fact that the optic cables can be wrapped around k times resulting into an "amplification" of the net effect:

$$
Physical Review & Research International, 3(3): 161-175, 2013
$$
\n
$$
\Delta T_{total\_SR} = k \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2}
$$
\n(2.13)

\nAssulting phase difference is:

\n
$$
\Delta S_{total\_SP} = c\Delta T_{total\_SP} = kc \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 k\omega}{c^2}
$$
\n(2.14)

The resulting phase difference is:

*Physical Review & Research International, 3(3): 161-175, 2013*\n
$$
\Delta T_{total\_SR} = k \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2}
$$
\n(2.13)

\nresulting phase difference is:

\n
$$
\Delta S_{total\_SR} = c \Delta T_{total\_SR} = kc \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 k \omega}{c}
$$
\n(2.14)

\n(2.14)

\n(2.15)

Physical Review & Research International, 3(3): 161-175, 2013<br>
(2.13)<br>  $4\pi R^2 \omega \over 2 - R^2 \omega^2 \approx \frac{4\pi R^2 k \omega}{c}$  (2.14)<br>  $\frac{\omega}{c}$ , "amplified" by the length of the fiber,  $L = 2\pi Rk$  and *Physical Review & Research International, 3(3): 161-175, 2013<br>*  $T_{total\_SR} = k \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2}$  *(2.13)*<br>
ulting phase difference is:<br>  $S_{total\_SR} = c\Delta T_{total\_SR} = k c \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 k \omega}{c}$  (2.14)<br>
the effect is Physical Review & Research International, 3(3): 161-175, 2013<br>
(2.13)<br>  $\frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 k \omega}{c}$  (2.14)<br>
n  $\frac{\omega}{c}$ , "amplified" by the length of the fiber,  $L = 2\pi Rk$  and<br>
.<br>
CORY OF THE FOG EXPERIMENT Physical Review & Research International, 3(3): 161-175, 2013<br>  $\Delta T_{total\_SR} = k \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2}$  (2.13)<br>
Seulting phase difference is:<br>  $\Delta S_{total\_SR} = c\Delta T_{total\_SR} = kc \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 k \omega}{c}$  (2.14)<br>
the effect that is, the effect is the first order in  $\frac{a}{c}$ , "amplified" by the length of the fiber<br> $\frac{c}{c}$ Physical Review & Research International, 3(3): 161-175, 2013<br>
(2.13)<br>  $\frac{4\pi R^2 \omega}{-R^2 \omega^2} \approx \frac{4\pi R^2 k \omega}{c}$  (2.14)<br>  $\frac{\omega}{c}$ , "amplified" by the length of the fiber,  $L = 2\pi Rk$  and<br>
PRY OF THE FOG EXPERIMENT<br>
isotrop by the radius of the gyroscope, *R* .

#### **2. THE MANSOURI-SEXL THEORY OF THE FOG EXPERIMENT**

Light speed is propagating with the isotropic speed  $c_0$  in the preferential frame. In the non preferential frame S associated with the center of the rotating FOG device light speed propagates at the speeds  $c_+$  in the direction of rotation and  $c_-$  in the direction against the rotation of the device (Fig. 2).





**Fig. 2. Detail of the experiment with anisotropic light speed**

where, for an infinitesimal angle of rotation  $d\phi$ :

$$
c_{\perp} \Delta t_{\perp} = R d\phi + \omega R \Delta t_{\perp} \tag{3.1}
$$

for the co-rotating front

$$
c_{\perp} \Delta t_{\perp} + \omega R \Delta t_{\perp} = R d \phi \tag{3.2}
$$

for the counter-rotating front.

$$
\Delta t_{+} = \frac{R d \phi}{c_{+} - \omega R} \tag{3.3}
$$

for the co-rotating front

$$
\Delta t_{-} = \frac{R d \phi}{c_{-} + \omega R} \tag{3.4}
$$

for the counter-rotating front. From (3.3) and (3.4) it follows that the phase difference element is:

$$
Physical Review & Research International, 3(3): 161-175, 2013
$$
\n
$$
\Delta s = c_{+} \Delta t_{+} - c_{-} \Delta t_{-} = R^{2} \omega d\phi \frac{(c_{-} + c_{+})}{(c_{+} + \omega R)(c_{+} - \omega R)}
$$
\n(a) (3.5) is a generalization of formula (21) in reference [8]. On the other hand,

Physical Review & Research International, 3(3): 161-175, 2013<br>  $^{2}\omega d\phi \frac{(c_{-}+c_{+})}{(c_{-}+\omega R)(c_{+}-\omega R)}$  (3.5)<br>
tion of formula (21) in reference [8]. On the other hand,<br>
d appears to be anisotropic in frame S, associated w Physical Review & Research International, 3(3): 161-175, 2013<br>  $(C_{-} + C_{+})$  ( $C_{+} + \omega R$ ) (3.5)<br>
( $C_{+} + \omega R$ ) $(C_{+} - \omega R)$  (3.5)<br>
of formula (21) in reference [8]. On the other hand,<br>
ears to be anisotropic in frame S, associ *Physical Review & Research International, 3(3): 161-175, 2013*<br> *s* =  $c_+ \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c_- + c_+)}{(c_- + \omega R)(c_+ - \omega R)}$  (3.5)<br>
(3.5)<br>
(3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
the *Physical Review & Research International, 3(3): 161-175, 2013*<br>  $(C_- + C_+)$  (3.5)<br>  $C_- + \omega R)(C_+ - \omega R)$  (3.5)<br>
formula (21) in reference [8]. On the other hand,<br>
ars to be anisotropic in frame S, associated with the<br>
at for sl Physical Review & Research International, 3(3): 161-175, 2013<br>  $\Delta s = c_+ \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c_- + c_+)}{(c_- + \omega R)(c_+ - \omega R)}$  (3.5)<br>
a (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
ng to Fig. 2 Physical Review & Research International, 3(3): 161-175, 2013<br>  $\frac{(c_{-}+c_{+})}{+\omega R)(c_{+}-\omega R)}$  (3.5)<br>
From (21) in reference [8]. On the other hand,<br>
to be anisotropic in frame S, associated with the<br>
for slow clock transport Formula (3.5) is a generalization of formula (21) in reference [8]. On the other hand, according to Fig. 2, light speed appears to be anisotropic in frame S, associated with the center of the rotating FOG, such that for slow clock transport synchronization and for *Physical Review & Research International, 3(3): 161-175, 2013*<br>  $\Delta s = c_+ \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c_- + c_+)}{(c_- + \omega R)(c_+ - \omega R)}$  (3.5)<br>
Formula (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
acco Physical Review & Research International, 3(3): 161-175, 2013<br>  $\Delta s = c_+ \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c_+ + c_+)}{(c_- + \omega R)(c_+ - \omega R)}$  (3.5)<br>
(3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
g to Fig. 2, Physical Review & Research International, 3(3): 161-175, 2013<br>  $c_x \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c_+ + c_+)}{(c_+ + \omega R)(c_+ - \omega R)}$  (3.5)<br>
5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
Fig. 2, light speed *Physical Review & Research International, 3(3): 161-175, 2013*<br>  $\Delta x = c_1 \Delta t_1 - c_2 \Delta t_- = R^2 \omega d\phi \frac{(c_1 + c_1)}{(c_2 + \omega R)(c_2 - \omega R)}$  (3.5)<br>
a (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
for 0 F Physical Review & Research International, 3(3): 161-175, 2013<br>  $\Delta s = c_1 \Delta t_1 - c_2 \Delta t_- = R^2 \omega d\phi \frac{(c_1 + c_1)}{(c_1 + \omega R)(c_1 - \omega R)}$  (3.5)<br>
(3.5)<br>
(3.5)<br>
(3.5)<br>
(3.5)<br>
(3.6)<br>
(5.6)<br>
(6.6)<br>
(6.7)<br>
(6.7)<br>
(6.7)<br>
(6.8)<br>
(6.8)<br>
(4.1-2*Physical Review & Research International. 3(3): 161-175, 2013<br>*  $s = c_+ \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c_- + c_+)}{(c_+ + \omega R)(c_+ - \omega R)}$  *(3.5)*<br>
(3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
(3.6) is a ge (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
g to Fig. 2, light speed appears to be anisotropic in frame S, associated with the<br>
f the rotating FOG, such that for slow clock transport sy Physical Review & Research International, 3(3): 161-175, 2013<br>  $c_x \Delta t_x = C \Delta t = R^2 \omega d \phi \frac{(c_1 + c_1)}{(c_1 + \omega R)(c_1 - \omega R)}$  (3.5)<br>
5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
Fig. 2, light speed app *Physical Review & Research International. 3(3): 161-175. 2013<br>*  $\Delta s = c_+ \Delta t_- - c_- \Delta t_- = R^2 \omega d\phi \frac{(c + c_1)}{(c + \omega R)(c_1 - \omega R)}$  *(3.5)*<br>
(3.5)<br>
(3.5)<br>
(3.5)<br>
(3.5)<br>
(3.5)<br>
(3.6)<br>
(4.7) a control of formula (21) in reference [8]. On *Physical Review & Research International, 3(3): 161-175, 2013*<br>  $\Delta s = c_+ \Delta t_+ - c_+ \Delta t_- = R^2 \omega d\phi \frac{(c_+ + c_+)}{(c_+ + \omega R)(c_+ - \omega R)}$  (3.5)<br>
a (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
eq in t *Physical Review & Research International, 3(3): 161-175, 2013<br>*  $s = c_1 \Delta t_1 - c_2 \Delta t = R^2 \omega d \phi \frac{(c_1 + c_1)}{(c_2 + \omega R)(c_2 - \omega R)}$  *(3.5)*<br>
(3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
g to Fig. 2, *Physical Review & Research International.* 3(3): 161-175, 2013<br>  $\Delta s = c_+ \Delta t_+ - c_- \Delta t_- = R^2 \omega d\phi \frac{(c + c_+)}{(c_- + \omega R)(c_+ - \omega R)}$  (3.5)<br>
Formula (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
econd  $-c_x\Delta t = R^2 \frac{\partial d}{\partial c_x} + \frac{c_1 - c_2}{\partial R}$  (3.5)<br>
a generalization of formula (21) in reference [8]. On the other hand,<br>
2, light speed appears to be anisotropic in frame S, associated with the<br>
alting FOG, such that for slow  $\Delta t_z = R^2 \omega d\phi \frac{(c_z + c_*)}{(c_z + \omega R)(c_z - \omega R)}$  (3.5)<br>
Ineralization of formula (21) in reference [8]. On the other hand,<br>
It speed appears to be anisotropic in frame S, associated with the<br>
FOG, such that for slow clock transpo  $c_x \Delta t_x - c_x \Delta t_x = R^2 \omega d \phi \frac{c - \omega_z t}{(c + \omega R)(c, -\omega R)}$  (3.5)<br>
(3.5)<br>
(3.5)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(2.8)<br>
(2.8)<br>
(2.7)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(3.6)<br>
(3.8)<br>
(3.6)<br>
(4.2*a*) cos( $\phi - \frac{\pi}{2}$ ) = 1 –  $\frac{v}{$  $\Delta s = c_z \Delta t_z - c_z \Delta t_z = R^2 \omega d\phi \frac{(c_z + c_z)}{(c_z + \omega R)(c_z - \omega R)}$  (3.5)<br>
a (3.5) is a generalization of formula (21) in reference [8]. On the other hand,<br>
and to Fig. 2. light speed appears to be anisotropic in frame S. associated wit  $t_{+} = c_{-}\Delta t_{-} = R^{2}\omega d\phi \frac{(c_{-} + c_{+})}{(c_{-} + \omega R)(c_{+} - \omega R)}$  (3.5)<br>
a a generalization of formula (21) in reference [8]. On the other hand,<br>
2. light speed appears to be anisotropic in frame S, associated with the<br>
idamy FOG

$$
\frac{c_+}{c_0} \approx 1 - \frac{v}{c_0} (1 + 2\alpha) \cos(\phi - \frac{\pi}{2}) = 1 - \frac{v}{c_0} (1 + 2\alpha) \sin \phi
$$
\n(3.6)

for the co-rotating front

$$
c_0 \t C_0
$$
\n
$$
c_0 \t C_0
$$
\n
$$
c_1 \t C_0
$$
\n
$$
c_2 \approx 1 - \frac{v}{c_0} (1 + 2\alpha) \cos(\phi + \frac{\pi}{2}) = 1 + \frac{v}{c_0} (1 + 2\alpha) \sin \phi
$$
\n
$$
c_0 \t counter-rotating front.
$$
\n
$$
E[\pi, 2\pi] \t c_+ \text{ and } c_- \text{ exchange roles. Substituting (3.6) and (3.7) into (3.5) we obtain:\n
$$
\Delta s \approx \frac{2R^2 \omega c_0 d\phi}{c_0^2 - [v(1 + 2\alpha) \sin \phi + \omega R]^2}
$$
\n
$$
d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
$$
\n
$$
d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
$$
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d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
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d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
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d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
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$$
d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
$$
\n
$$
d = \frac{2R^2 \omega c_0 d\phi}{\omega^2}
$$
\n
$$
d = \frac{2R^2 \omega c_0 d\phi}{\
$$
$$

for the counter-rotating front.

and *c* exchange roles. Substituting (3.6) and (3.7) into (3.5) we obtain:

$$
\Delta s \approx \frac{2R^2 \omega c_0 d\phi}{c_0^2 - \left[v(1 + 2\alpha)\sin\phi + \omega R\right]^2}
$$
(3.8)

The total phase differential between the two light paths obtained through the integration of the phase difference element is:

counter-rotating front.  
\n
$$
E[\pi, 2\pi] c_+
$$
 and  $c_-$  exchange roles. Substituting (3.6) and (3.7) into (3.5) we obtain:  
\n
$$
\Delta s \approx \frac{2R^2 \omega c_0 d\phi}{c_0^2 - [v(1 + 2\alpha)\sin \phi + \omega R]^2}
$$
(3.8)  
\ntal phase differential between the two light paths obtained through the integration of  
\n
$$
\Delta S_{total\_MS}(v, \alpha, \omega) \approx 4R^2 \omega c_0 k_0^{\frac{\pi}{2}}
$$

$$
\frac{d\phi}{c_0^2 - [v(1 + 2\alpha)\sin \phi + \omega R]^2}
$$
(3.9)  
\n
$$
\omega bling of the integral (3.9) is caused by c_+
$$
 and  $c_-$  exchanging roles in the interval  
\n
$$
2\pi 1
$$
 Using the notation  $A(v, \alpha) = v(1 + 2\alpha) (A_0)$  is a function of  $v$  and  $\alpha$ .) and

(a)  $\frac{v}{c_0}(1+2\alpha)\cos(\phi-\frac{\pi}{2})=1-\frac{v}{c_0}(1+2\alpha)\sin\phi$  (3.6)<br>
ng front<br>  $\frac{v}{c_0}(1+2\alpha)\cos(\phi+\frac{\pi}{2})=1+\frac{v}{c_0}(1+2\alpha)\sin\phi$  (3.7)<br>
rotating front.<br>  $\left.\int_{c_+}^2 \cot(\phi-\frac{\pi}{2})\cos(\phi+\frac{\pi}{2})=1+\frac{v}{c_0}(1+2\alpha)\sin\phi\right]$  (3.7)<br>
rotating front  $\epsilon \approx 1 - \frac{v}{c_0} (1 + 2\alpha) \cos(\phi - \frac{\pi}{2}) = 1 - \frac{v}{c_0} (1 + 2\alpha) \sin \phi$  (3.6)<br>
crotating front<br>
crotating front<br>  $\approx 1 - \frac{v}{c_0} (1 + 2\alpha) \cos(\phi + \frac{\pi}{2}) = 1 + \frac{v}{c_0} (1 + 2\alpha) \sin \phi$  (3.7)<br>
(3.7)<br>
canner-rotating front.<br>  $\pi$ , 2x] *c*, a *S*<sup>1</sup>  $\frac{1}{6}$  is following holds by (1.2):<br>  $\frac{1}{6} \times 1 - \frac{V}{c_0} (1 + 2\alpha) \cos(\phi - \frac{\pi}{2}) = 1 - \frac{V}{c_0} (1 + 2\alpha) \sin \phi$  (3.6)<br>  $\frac{1}{6} \sin \left(\frac{\pi}{6}\right) = \frac{1}{c_0} (1 + 2\alpha) \cos(\phi + \frac{\pi}{2}) = 1 + \frac{V}{c_0} (1 + 2\alpha) \sin \phi$  (3.7)<br>  $\frac{1}{\sin \left(\frac{\$  $\frac{\pi}{2}$ ) = 1 -  $\frac{v}{c_0}$  (1+2 $\alpha$ ) sin  $\phi$  (3.6)<br> **c** (3.6)<br> **c**  $\frac{\pi}{2}$ ) = 1 +  $\frac{v}{c_0}$  (1+2 $\alpha$ ) sin  $\phi$  (3.7) into (3.5) we obtain:<br> **c**  $\alpha R$ <sup>2</sup> (3.8)<br> **c**  $\alpha R$ <sup>2</sup> (3.8)<br> **c**  $\alpha R$ <sup>2</sup> (3.8)<br> **c**  $\alpha R$ <sup>2</sup> ( ing to rg. 2, ingits speed appears to be anisotoply in manner 5, associated wind the<br>
ord the rotating FOG, such that for slow clock transport synchronization and for<br>  $\frac{c_+}{c_0} \approx 1 - \frac{v}{c_0} (1 + 2\alpha) \cos(\phi - \frac{\pi}{2}) = 1 - \frac{$ sion that for slow clock transport synchronization and for<br>  $(1.2)$ :<br>  $\begin{aligned}\n&\frac{\pi}{2} = 1 - \frac{v}{c_0} (1 + 2\alpha) \sin \phi \qquad (3.6) \\
&\frac{\pi}{2} = 1 + \frac{v}{c_0} (1 + 2\alpha) \sin \phi \qquad (3.7) \\
&\frac{\pi}{2} = 1 + \frac{v}{c_0} (1 + 2\alpha) \sin \phi \qquad (3.7) \\
&\frac{\pi}{2} = 1 + \frac{v}{c$ The doubling of the integral (3.9) is caused by  $c_+$  and  $c_-$  exchanging roles in the interval  $\frac{d}{c_0} = 1 - \frac{d}{c_0} (1 + 2a) \cos(\phi - \frac{1}{2}) = 1 - \frac{1}{c_0} (1 + 2a) \sin \phi$  (3.6)<br>
for the co-rotating front<br>  $\frac{c}{c_0} \approx 1 - \frac{v}{c_0} (1 + 2a) \cos(\phi + \frac{\pi}{2}) = 1 + \frac{v}{c_0} (1 + 2a) \sin \phi$  (3.7)<br>
for the counter-rotating front.<br>
For  $\phi \in [\$ for the co-rotating front<br>  $\frac{c_2}{c_0} \approx 1 - \frac{v}{c_0} (1 + 2\alpha) \cos(\phi + \frac{\pi}{2}) = 1 + \frac{v}{c_0} (1 + 2\alpha) \sin \phi$  (3.7)<br>
for the counter-rotating front.<br>
For  $\phi \in [\pi, 2\pi]$  c, and c exchange roles. Substituting (3.6) and (3.7) into (3. two light paths obtained through the integration of<br>  $\frac{d\phi}{d(1+2\alpha)\sin\phi + \omega R]^2}$  (3.9)<br>
sed by  $c_+$  and  $c_-$  exchanging roles in the interval<br>  $z) = v(1+2\alpha) (A \text{ is a function of } v \text{ and } \alpha)$  and<br>  $\frac{1}{z^2 - A^2} + \frac{1}{\sqrt{(B + c_0)^2 - A^2}}$  (3 (1+2 $\alpha$ ) cos( $\phi + \frac{\pi}{2}$ ) = 1+ $\frac{V}{C_0}$ (1+2 $\alpha$ ) sin  $\phi$  (3.7)<br>
ting front.<br>
4 and c<sub>c</sub> exchange roles. Substituting (3.6) and (3.7) into (3.5) we obtain:<br>  $2R^2 \alpha C_0 d\phi$  ( $v(1+2\alpha) \sin \phi + \omega R$ ]<sup>2</sup> (3.8)<br>
fferential bet 2  $c_0$ <br>
change roles. Substituting (3.6) and (3.7) into (3.5) we obtain:<br>  $\frac{\phi}{\phi + \omega R]^2}$  (3.8)<br>
ween the two light paths obtained through the integration of<br>  $\left(\frac{d\phi}{d\omega}\right)^2$ <br>  $\left(\frac{d\phi}{d\omega^2 - [1(1+2\alpha)\sin\phi + \omega R]^2}\right)$  (  $\frac{1}{C_0} = 1 - \frac{1}{C_0} (1 + 2\alpha) \cos(\phi + \frac{1}{2}) = 1 + \frac{1}{C_0} (1 + 2\alpha) \sin \phi$  (3.7)<br>
counter-rotating front.<br>  $\int_{C_0} = \left[ \pi, 2\pi \right] c_+$  and  $c_-$  exchange roles. Substituting (3.6) and (3.7) into (3.5) we obtain:<br>  $\int_{C_0} = \frac{2R^$ *B* and the substituting (3.6) and (3.7) into (3.5) we obtain:<br>  $\frac{\partial}{\partial t} + \omega R_1^2$  (3.8)<br>
een the two light paths obtained through the integration of<br>  $\int_0^t \frac{d\phi}{c_0^2 - [1(1+2\alpha)\sin\phi + \omega R_1^2]}$  (3.9)<br>
is caused by  $c_x$  a  $\frac{c_-}{c_0} \approx 1 - \frac{v}{c_0} (1 + 2\alpha)\cos(\phi + \frac{\pi}{2}) = 1 + \frac{v}{c_0} (1 + 2\alpha)\sin\phi$  (3.7)<br> **e** counter-rotating front.<br>  $\Delta \varepsilon \approx \frac{2R^2 \omega c_u d\phi}{c_0^2 - [v(1 + 2\alpha)\sin\phi + \omega R]^2}$  (3.6) and (3.7) into (3.5) we obtain:<br>  $\Delta s \approx \frac{2R^2 \omega c_u d\phi}{c_0$  $\frac{\pi}{2}$ ) = 1 +  $\frac{v}{c_0}$  (1+2 $\alpha$ ) sin  $\phi$  (3.7)<br>
onge roles. Substituting (3.6) and (3.7) into (3.5) we obtain:<br>  $+\omega R$ <sup>2</sup> (3.8)<br>
on the two light paths obtained through the integration of<br>  $\frac{d\phi}{c_0^2 - [v(1+2\alpha)\sin\phi$ for the counter-rotating front.<br>
For  $\phi \in [1, 2, \pi]$  c, and c, exchange roles. Substituting (3.6) and (3.7) into (3.5) we obtain:<br>  $\Delta s \approx \frac{2R^3 \alpha c_0 d\phi}{c_0^2 - |v(1 + 2\alpha) \sin \phi + \omega R|^2}$  (3.8)<br>
The total phase difference eigen g of the integral (3.9) is caused by  $c_+$  and  $c_-$  exchanging roles i<br>
. Using the notation  $A(v, \alpha) = v(1+2\alpha)$  (*A* is a function of variable).<br>
. Ms  $\approx 2\pi R^2 \omega k \left(\frac{1}{\sqrt{(B-c_0)^2 - A^2}} + \frac{1}{\sqrt{(B+c_0)^2 - A^2}}\right)$ <br>
ty check sh (3.8)<br>
the set of  $\frac{2\pi}{\pi r^2}$  (3.4)<br>
the phase difference element is:<br>  $\int_{\text{total}}^{\infty} \frac{d\phi}{d\phi} d\phi = 0$ <br>  $\int_{\text{total}}^$  $\Delta s \approx \frac{2R^2 \alpha c_u d\phi}{c_0^2 - [v(1 + 2\alpha)\sin\phi + \omega R]^2}$  (3.8)<br>
al phase differential between the two light paths obtained through the integration of<br>
se difference element is:<br>  $\Delta S_{\text{new} \rightarrow \text{so}}(v; \alpha, \omega) \approx 4R^2 \alpha c_0 k_0^2$ <br>  $\Delta S_{$ 2*K*  $\alpha_{c_0}$ (3.8)<br>  $\left[\frac{\sqrt{1+2\alpha_0}\sin\phi + \omega R\right]^2}{\sqrt{1+2\alpha_0}\sin\phi + \omega R\left(\frac{\pi}{2}\right)}$  (3.8)<br>
(3.8)<br>
c e element is:<br>  $\left[\cos\left(\frac{\pi}{2}\right)\right]$   $\left[\cos\left(\frac{\pi}{2}\right)$   $\left[\cos\left(\frac{\pi}{2}\right)\right]$   $\left[\cos\left(\frac{\pi}{2}\right)\right]$  (3.9)<br>
c e integral (3.9) is cau  $\Delta s \approx \frac{2R^2 \alpha c_0 d\phi}{c_0^3 - [v(1 + 2\alpha)\sin\phi + \omega R]^2}$  (3.8)<br>
As  $\approx \frac{2R^2 \alpha c_0 d\phi}{c_0^3 - [v(1 + 2\alpha)\sin\phi + \omega R]^2}$  (3.8)<br>
and phase differential between the two light paths obtained through the integration of<br>  $\Delta S_{\text{net},j,6}(v;\alpha,\omega$ 

$$
\Delta S_{total_{M}} = 2\pi R^2 \omega k \left( \frac{1}{\sqrt{(B - c_0)^2 - A^2}} + \frac{1}{\sqrt{(B + c_0)^2 - A^2}} \right) \tag{3.10}
$$

known SR expression (2.7):

$$
\Delta S_{total\_SR} = \frac{4\pi R^2 \omega k c_0}{c_0^2 - (\omega R)^2}
$$
\n(3.11)

168

That is, in SR the phase difference is independent of the speed between the lab and the *Physical Review & Research International, 3(3):* 161-175, 2013<br>
That is, in SR the phase difference is independent of the speed between the lab and the<br>
"preferential" frame  $\Sigma$ . Given that  $v \ll c_0$ , the effect is very devices. The difference:

$$
\Delta S_{\text{violation}} = \Delta S_{\text{total}} \Delta S_{\text{total}} - \Delta S_{\text{total}} \Delta S_{\text{R}}
$$
\n(3.12)

*Physical Review & Research International, 3(3): 161-175, 2013*<br> **s**, in SR the phase difference is independent of the speed between the lab and the<br>
tential" frame  $\Sigma$ . Given that  $\nu \ll c_0$ , the effect is very close to is the actual Mansouri-Sexl violation expressed in terms of fraction of a fringe (in  $\mu$ m) and it is a function of the Mansouri-Sexl parameter  $\alpha$ , the angular speed  $\omega$  of the FOG with respect to the lab frame S and the relative speed of the lab  $v$  with respect to the preferential frame  $\Sigma$ . As it can be seen from (3.9), any deviation from -0.5 for the parameter  $\alpha$  attracts a dependency of the result in terms of the speed  $v$  between S and  $\Sigma$ . As opposed to the case of the SR formula (2.12), the Mansouri-Sexl formula (3.10) depends on the refraction index via <sup>0</sup> *c c n* / restricting the constraining of the parameter to experiments that must use fiber optics with refraction indexes close to unity. The difference is due to the fact that formulas (3.6) and (3.7) are just approximations in the Mansouri-Sexl theory whereas formula (2.9) is exact. In our experiment, we made use of the above prediction in order to set constrains on light speed anisotropy. *Physical Review & Research International, 3(3): 161-175, 2013*<br>
That is, in SR the phase difference is independent of the speed between the lab and the<br>
"preferential" frame  $\Sigma$ . Given that  $v \ll c_0$ , the effect is very Physical Review & Research International, 3(3): 161-175, 2013<br>
That is, in SR the phase difference is independent of the speed between the lab and the<br>
cyreferential" frame  $\Sigma$ . Given that  $v \ll c_0$ , the effect is very cl That is, in SR the phase difference is independent of the speed between the lab and the "preferential" frame  $\Sigma$ . Given that  $v \ll c_6$ , the effect is very close to null for non-rotating devices. The difference:<br>  $\Delta S_{\text{scalar$ That is, in SR the phase difference is independent of the speed between the lab and the<br>
Typeferential" frame 2. Given that  $v \ll c_k$ , the effect is very close to null for non-rotating<br>
devices. The difference:<br>  $N_{\text{volume}} = N_{$ 3.12)<br>
S<sub>ood\_38</sub> (3.12)<br>
Sood\_38<br>
Sood\_38<br>
Sood\_38<br>
Sood\_38<br>
1.12)<br>
Sood\_38<br>
1.12)<br>
Sood\_39<br>
1.12)<br>
Sood\_39<br>
1.12 (1.13)<br>
4.12 (1.13)<br>
Any deviation from 1.0.5 for the parameter *C* attended<br>
in terms of the parameter *C*  $\Delta_{\text{CALM}} = \Delta S_{\text{CAL}}$ ,  $\Delta S_{\text{CAL}}$  with a cost violation expressed in terms of fraction of a finge (in um) and it control of the Mansouri-Sext violation expressed in terms of fraction of a finge (in um) and it columina actual Mansouri-Sexl violation expressed in terms of fraction of a fringe (in <sub>i</sub>m) and it<br>anction of the Mansouri-Sexl parameter  $\alpha$ , the angular speed  $\alpha$  of the FOG with<br>the tot the lab frame S and the relative speed *o* is a direction of the Marison-Cox indication (syncocuri curring that the FOG with respect to the lab frame S and the relative speed of the BTOG with respect to the lab frame S and the relative speed of the PTOG with r

was executed (latitude 37<sup>o</sup>52'18" N)  $v_d = 0.355$ km / s. Finally,  $v_r = \omega R$  is the active rotation speed of the FOG so:

$$
v(t) = v_s + v_e \sin[\Omega_v(t - t_0)] \cos \Phi_E + v_d \sin[\Omega_d(t + t_d)] \cos \Phi_A + v_r \sin(\omega t) \cos \Phi_B \tag{3.13}
$$

Here  $\Phi_{\scriptscriptstyle{A}} \approx 8^{\circ}$  is the angle between the equatorial plane and the velocity of the sun.  $6<sup>o</sup>$  is the declination between the plane of Earth's orbit and the velocity of the Sun, espect to the lab frame S and the relative speed of the lab v with respect to the preferentiation<br>rame  $\Sigma$ . As it can be seen from (3.9), any deviation from -0.5 for the parameter  $\alpha$  attracta dependency of the result i  $\Phi_R \approx 33^\circ$  is the declination between the plane of FOG plane and the velocity of the Sun, case of the SR tormula (2.12), the Mansour-Sexi formula (3.10) depends on the retraction<br>ndex via  $c_n = c/n$  restricting the constraining of the parameter  $\alpha$  to experiments that must<br>sue fiber optics with refraction indexe and the SR formula (2.12), the Mansouri-on-vision of the speed in the speed of the sect of the SR formula (2.12), the Mansouri-Sear of the SR formula (3.10) depends on the effection<br>developed on the SR formula (2.12), the 1 sidereal day,  $t_0$  and  $t_d$  are determined by the phase and case of the SR formula (2.12), the Mansour-Seal formula (3.10) depends on the refaction<br>index via  $c_n = c/n$  restricting the constraining of the parameter  $\alpha$  to experiments hat must<br>use fiber optics with refraction indexes *v* will be reflected in the phase difference (3.10). In other words, the phase difference (3.10) will exhibit a characteristic time signature when measured over a sufficiently long time. In order to constrain the parameter  $\alpha$  we will take a series of measurements at different angular speeds  $\omega$  over periods of time long enough such that we could integrate the The laboratory velocity  $v(t)$  has contributions [10-19] from the motion of the Sun with<br>respect to frame  $\Sigma$  with a constant velocity  $v_x = 377km/s$ , while Earth's orbital motion<br>around the Sun  $v_x = 30km/s$ . For example, in t enables us to further simplify expression (3.10) and, subsequently, (3.12) by using Taylor expansion such that we can express the translational effects in  $v$  in a simpler form:

*Physical Review & Research International, 3(3): 161-175, 2013*\n
$$
\Delta S_{total\_MS} \approx \frac{4\pi R^2 k\omega}{c_0} + 4\pi R^2 k\omega (1+2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2 (1+2\alpha) \frac{(2\pi R k\omega)v}{c^2}
$$
\n(3.14)

\namplification of the effect due to the presence of large values of the coil number *k* is a e change since we can achieve  $2\pi R k\omega >> v$  for suitable optical cable lengths even.

Physical Review & Research International, 3(3): 161-175, 2013<br>  $\frac{d^2k\omega}{dr^2} + 4\pi R^2 k\omega (1+2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 kn)\omega}{c} + 2Rn^2(1+2\alpha)\frac{(2\pi Rk\omega)v}{c^2}$  (3.14)<br>
the effect due to the presence of large values of the coil Physical Review & Research International, 3(3): 161-175, 2013<br>  $\frac{4\pi R^2 k\omega}{c_0} + 4\pi R^2 k\omega(1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k\eta)\omega}{c} + 2Rn^2(1 + 2\alpha)\frac{(2\pi Rk\omega)v}{c^2}$  (3.14)<br>
plification of the effect due to the presence of lar *Physical Review & Research International, 3(3): 161-175, 2013*<br>  $S_{\text{total\_MS}} \approx \frac{4\pi R^2 k \omega}{c_0} + 4\pi R^2 k \omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha) \frac{(2\pi R k \omega)v}{c^2}$  (3.14)<br> *Applification of the effect due to the* Physical Review & Research International, 3(3): 161-175, 2013<br>  $\frac{R^2 k\omega}{c_0} + 4\pi R^2 k\omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha) \frac{(2\pi R k\omega)v}{c^2}$  (3.14)<br>
the effect due to the presence of large values of the c *Physical Review & Research International, 3(3): 161-175, 2013<br>*  $\Delta S_{total\_MS} \approx \frac{4\pi R^2 k\omega}{c_0} + 4\pi R^2 k\omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha)\frac{(2\pi R k\omega)v}{c^2}$  *(3.14)<br>
amplification of the effect due to the pre* The amplification of the effect due to the presence of large values of the coil number *k* is a Physical Review & Research International, 3(3): 161-175, 2013<br>  $\Delta S_{\text{total\_MS}} \approx \frac{4\pi R^2 k \omega}{c_0} + 4\pi R^2 k \alpha (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha) \frac{(2\pi R k \omega)v}{c^2}$  (3.14)<br>
The amplification of the effect due to with moderate angular speeds of rotating the FOG. In his analysis, made 14 years ago, Stedman [5] expressed pessimism that FOGs can be used in the detection of light speed anisotropy but FOGs have made huge advancements in the past decade, not only in terms of precision but also in terms of the fiber optic length. For example, in our experimental Physical Review & Research international, 3(3): 161-175, 2013<br>  $\Delta S_{\text{mod}} = 4\pi R^2 k \omega + 4\pi R^2 k \omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c_0^2} + 2Rn^2(1 + 2\alpha) \frac{(2\pi R k \omega)v}{c^2}$  (3.14)<br>
The amplification of the effect due to the pr measurements at different angular speeds  $\omega$  over periods of time long enough such that we could integrate the sinusoidal effects shown in (3.13). Substituting (3.13) into (3.14) we obtain: Physical Review & Research International, 3(3): 161-175, 2013<br>  $\Delta S_{\text{mod-3N}} \approx \frac{4\pi R^2 k \omega}{c_0} + 4\pi R^2 k \omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha) \frac{(2\pi R k \omega)v}{c^2}$  (3.14)<br>
mplification of the effect due to the giant contained the FOG. In his analysis, made 14 years agency with moderate angular speeds of rotating the FOG. In his analysis, made 14 years agency but FOGs have made lugared the contrain the past decade, not only in t Physical Review & Research International, 3(3): 161-175, 2013<br>  $\frac{k\omega}{4\pi R^2 k\omega(1+2\alpha)} \frac{v}{c_0^2} = \frac{(4\pi R^2 k\pi)\omega}{c} + 2Rn^2(1+2\alpha)\frac{(2\pi R k\omega)v}{c^2}$  (3.14)<br>
e effect due to the presence of large values of the coil number k i *Physical Review & Research International, 3(3): 161-175, 2013<br>*  $\frac{4\pi R^2 k \omega}{c_0} + 4\pi R^2 k \omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha) \frac{(2\pi R k \omega)v}{c^2}$  *(3.14)<br>
on of the effect due to the presence of large value* Physical Review & Research International, 3(3): 161-175, 2013<br>  $R^2 k\omega (1+2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 \sin)\omega}{c} + 2Rn^2(1+2\alpha) \frac{(2\pi Rk\omega)v}{c^2}$  (3.14)<br>
due to the presence of large values of the coil number k is a<br>
achieve  $2\pi R$ Physical Review & Research International, 3(3): 161-175, 2013<br>  $T^2 k\omega(1+2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 kn)\omega}{c} + 2Rn^2(1+2\alpha)\frac{(2\pi Rk\omega)v}{c^2}$  (3.14)<br>
thue to the presence of large values of the coil number k is a<br>
nef rotating t

$$
\Delta S_{total, MS} = C_0 + C_{11} \sin(\Omega_v t) + C_{12} \cos(\Omega_v t) + C_{21} \sin(\Omega_d t) + C_{22} \cos(\Omega_d t) + C_3 \sin(\omega t)
$$
\n(3.15)

*Physical Review & Research International*. 3(3): 161-175, 2013  
\n
$$
\Delta S_{\text{local\_MS}} = \frac{4\pi R^2 k\omega}{c_0} + 4\pi R^2 k\omega (1 + 2\alpha) \frac{v}{c_0^2} = \frac{(4\pi R^2 k n)\omega}{c} + 2Rn^2(1 + 2\alpha) \frac{(2\pi Rk\omega)v}{c^2}
$$
\n(3.14)  
\nThe amplification of the effect due to the presence of large values of the coil number k is a  
\ngame change since we can achieve 2\pi Rk\omega > v for suitable optical scale length seven  
\nwith moderate angular speeds of rotating the FOGs. In his analysis, made 14 years ago,  
\nSiledmn [5] expressed pessim that FOGs can be used in the effect of of light speed  
\nanisotropy but FOGs have made huge advancesments in the past detected, not only in terms  
\nseting, 2\pi Rk\omega = 1200m. In order to constrain the parameter  $\alpha$  we will take a series of  
\noutal integral the sinusoidal effects shown in (3.13). Substituting (3.13) into (3.14) we  
\nobtain:  
\n
$$
\Delta S_{\text{test\_MS}} = C_v + C_{11} \sin(\Omega_1 t) + C_{12} \cos(\Omega_1 t) + C_{13} \sin(\Omega_n t) + C_{21} \cos(\Omega_n t) + C_{23} \sin(\Omega_n t)
$$
\n(3.15)  
\nwhere  
\n
$$
\frac{C_v}{C_v} = \frac{\frac{4\pi R^2 k \omega m}{c} + \frac{4\pi R^2 k \omega m^2}{c} (1 + 2\alpha) \frac{v}{c}}{c}
$$
\n
$$
\frac{C_v}{C_v} = \frac{\frac{4\pi R^2 k \omega m}{c} (1 + 2\alpha) \frac{v}{c} \sin(\Omega_1 t) \cos \Phi_2}{c}
$$
\n(3.16)  
\nwhere  
\n
$$
\frac{C_v}{C_v} = \frac{\frac{4\pi R^2 k \omega m}{c} (1 + 2\alpha) \frac{v}{c} \sin(\Omega_1 t) \cos \Phi_2}{c}
$$
\n(3.17)  
\nwhere  
\n
$$
\frac{C_v}{C_v} = \frac{\frac{4\pi R^2 k \omega m^2}{c} (1 + 2\alpha) \frac{v}{c} \sin(\Omega_1 t) \cos \Phi_
$$

 $2l_{z}$ *c* observation is that the coefficient  $\,C_{0}\,$  is much larger than the other coefficients in the Fourier expansion.

#### **3. THE EXPERIMENTAL SETUP AND THE RESULTS**

We used two experimental setups, both based on commercially available FOGs. One uses from Emcore Inc mounted on a Yaskawa SGMJV Sigma-5 high precision turntable with variable angular speed (Fig. 3). We made four sets of measurements, alternating between the two FOGs, in a 24 hr interval, at 6 hours intervals in order to best capture the effects of the variation of the Earth speed expressed in (3.14) as well as the diurnal changes of temperature affecting the FOGs. Each set of measurements is composed of 10 runs, labeled 0-9. We repeated the measurements sets four times, at different angular speeds, varying from  $\omega = 30$  to  $\omega = 150$ . The  $\Delta S_{\text{violation}}$  measurements, expressed in  $\mu$ m, including the calculation of the error bars, are presented in Tables 1 through 4.

$\omega = 30$			2		3		4	5	6		7	8
Series1	0.4186667		0.418667		0.4186667		0.4186667	0.41866693	0.41866676	0.4186666		0.41985027
Series2	0.4186668		0.418665		0.4186667		0.4186667	0.41866671	0.418666667	0.4186666		0.41985067
Series3	0.4186666		0.418667		0.41866672		0.4186733	0.41866672	0.418666677	0.4186667		0.41985036
Series4	0.4186668		0.418666		0.41866668		0.4186667	0.41866673	0.418666668	0.4186666		0.41985304
<b>STD DEV</b>	7.414E-08		5.27E-07		2.1344E-08		3.313E-06	1.0722E-07	4.49526E-08		$3.2E-08$	1.3136E-06
	STD ERR 1.853E-08		1.32E-07		5.3359E-09		8.283E-07	2.6805E-08	1.12382E-08		8.001E-09	3.284E-07
	0.4250000											
	0.4230000											
												$\triangle$ Series1
$\Delta S_{\rm violation}$	0.4210000											
										n		■ Series2
												Series3
	0.4190000			y	Ā	Ā	Ā	Ā	Ā Ā			Series4
	0.4170000											
	0.4150000											
		0			$\overline{2}$	3	4	5	6 7	8	9	
								Run				

Table 1.  $\Delta S$ <sub>violation</sub> measurements at 6 am

Table 2.  $\Delta S$ <sub>violation</sub> measurements at 12 pm

$0=50$			4	6	8
	Series1 0.6977778 0.697778 0.6977778			0.6977778 0.69777822 0.697777933 0.6977777 0.69775282	
	Series2 0.6977780 0.697776 0.6977778			0.6977779 0.69777784 0.697777778 0.6977777 0.69775289	
				Series3 10.6977777 10.697778 10.69777787 10.6977789 10.69777787 10.697777796 10.6977778 10.69775284	
				Series4 10.6977780 10.697777 10.6977778 10.6977779 10.69777789 10.697777779 10.6977777 10.69775306	
				STD DEV 1.236E-07 8.79E-07 3.5573E-08 5.232E-07 1.787E-07 7.4921E-08 5.334E-08	1.0972E-07
				STD ERR 3.089E-08 2.2E-07 8.8932E-09 1.308E-07 4.4675E-08 1.87303E-08 1.333E-08 2.7431E-08	



171

$\omega = 100$		2	3	4	5	6		8
Series1	1.3955556	1.395555	1.3955556	1.3955556	1.39555644	1.395555867	1.3955553	1.39550564
Series2	1.3955559	1.395552	1.3955556	1.3955557	1.39555569	1.395555556	1.3955553	1.39550573
Series3	1.3955555	1.395555	1.39555573	1.3955578	1.39555573	1.395555591	1.3955556	1.39550568
Series4	1.3955560	1.395554	1.3955556	1.3955557	1.39555578	1.395555559	1.3955555	1.39550572
<b>STD DEV</b>	2.471E-07	1.76E-06	7.1146E-08	1.046E-06	3.574E-07	1.49842E-07	1.067E-07	4.1595E-08
	STD ERR 6.178E-08	4.4E-07	1.7786E-08	2.616E-07	8.9351E-08	3.74605E-08	2.667E-08	1.0399E-08
1.3957500								
1.3957000								
1.3956500								
1.3956000								$\triangle$ Series1
1.3955500		Ŷ	주 Ŷ	垒	ņ P	Ÿ		■ Series2
								Series3
1.3955000							m	Series4
1.3954500								
1.3954000								
1.3953500								
	0		$\overline{c}$ 3	4	5 6	7	8 9	

Table 3.  $\Delta S$ <sub>violation</sub> measurements at 6 pm

Table 4.  $\Delta S_{\text{violation}}$  measurements at 12 am

$\omega = 150$		2	3	4	5	6	7	8
Series1	2.0933333		2.0933334	2.0933334	2.09333467	2.0933338	2.0933330	2.09325913
2.0933339 Series2		2.093327	2.0933334	2.0933336	2.09333353	2.093333334	2.093333	2.09325927
Series3	2.0933332	2.093333	2.0933336	2.0933367	2.0933336	2.093333387	2.0933333	2.09325918
Series4	2.0933339	2.093331	2.0933334	2.0933336	2.09333367	2.093333338	2.0933332	2.0932600
<b>STD DEV</b>	3.707E-07	2.64E-06	1.067E-07	1.57E-06	5.361E-07	2.24763E-07	1.6E-07	4.0634E-07
<b>STD ERR</b>	9.267E-08	6.59E-07	2.668E-08	3.924E-07	1.3403E-07	5.61908E-08	4E-08	1.0158E-07
2.0940000								
2.0939000								
2.0938000								
2.0937000								
								$\triangle$ Series1
2.0936000								Series2
2.0935000								
2.0934000								Series3
				Ÿ				$\times$ Series4
2.0933000		Ŧ	햑 Ŧ		Ω e	e	I m	
2.0932000								
2.0931000								
2.0930000								
	0		$\overline{c}$ 3	4	5 6	7	8 9	

Based on the measurements we developed a best fit approximation in the form of a Fourier

$$
\hat{\Delta S} = \hat{C}_0 + \hat{C}_{11} \sin(\Omega_y t) + \hat{C}_{12} \cos(\Omega_y t) + \hat{C}_{21} \sin(\Omega_d t) + \hat{C}_{22} \cos(\Omega_d t) + \hat{C}_3 \sin(\omega t)
$$
 (4.1)

Physical Review & Research International, 3(3): 161-175, 2013<br>Based on the measurements we developed a best fit approximation in the form of a Fourier<br>expansion:<br> $\Delta S = \hat{C}_0 + \hat{C}_{11} \sin(\Omega_x t) + \hat{C}_{22} \cos(\Omega_x t) + \hat{C}_{21} \sin(\Omega_x t)$ The standard error in the determination of  $C_0$  is equal to  $1.33 \times 10^{-14}$ . Comparing with (3.16) al Review & Research International, 3(3): 161-175, 2013<br>
est fit approximation in the form of a Fourier<br>  $\frac{\sin(\Omega_d t) + C_{22} \cos(\Omega_d t) + C_3 \sin(\omega t)}{(4.1)^2}$ <br>
is equal to 1.33×10<sup>-14</sup>. Comparing with (3.16)<br>
so of the FOGs employed, and taking into considerations the characteristics of the FOGs employed, this results into a constraint of  $\alpha + 0.5 \times (1.2 \pm 0.83) \times 10^{-6}$  for the parameter  $\alpha$ .



**Fig. 3. The experimental setup**

The measurement errors can be attributed in totality to the systematic errors introduced by the FOG devices and the turntable, better results will be obtained in the next generation of the experiments when more precise FOGs become available and we can get a better control over maintaining constant angular speed of the underlying turntable.

#### **3.1. FUTURE WORK AND COMPARISON WITH OTHER METHODS**

Presently, the method using FOGs results into lesser constraints than the methods using resonating cavities [10-19]. On the other hand, our results are better by an order of magnitude than the ones of Champeney et al. [20] while being one order of magnitude less restrictive than the experiment executed by Isaak [21]. We plan to repeat the measurements as higher precision FOGs become available. The nice aspect about using FOGs is that they have no moving parts and that their technology is advancing very quickly, both reasons for increasing precision over time. Thus, we can put ever tightening constraints over the Mansouri-Sexl parameter  $\alpha$  using commercially available equipment that costs a fraction of the price of the specially designed equipment for such kind of experiments.

#### **4. CONCLUSION**

We have developed the Mansouri-Sexl theory for the FOG experiment. We have shown that the Mansouri-Sexl violation is a function of the Mansouri-Sexl parameter  $\alpha$ , the angular speed  $\omega$  and of the relative speed of the lab v with respect to the preferential frame  $\Sigma$ . We have shown how the FOG experiment can be used in order to detect light speed anisotropy within the framework of the Mansouri-Sexl theory and we constrained the parameter  $\alpha$  to Physical Review & Research International, 3(3): 161-175, 2013<br>within the framework of the Mansouri-SexI theory and we constrained the parameter  $\alpha$  to<br>less than  $-0.5 \pm 0.83 \times 10^{-6}$ .<br>**AKNOWLEDGEMENTS**<br>The author is grate

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#### **COMPETING INTERESTS**

Author has declared that no competing interests exist.

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