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On Finding Geodesic Equation of Two Parameters Extreme Value and Related Six Distribution

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper we use Darboux's theory to set up a second order partial differential equation. Later, we will use the variable transformation method to rotate the axis, $by\ 22.87562354\ \text{degree}$, in order to remove the interaction terms, which will allow us to find the geodesic equation of two parameter's extreme value distribution. We also list and prove some useful moments of this distribution. Finally, we apply six transformations that relate this extreme value distribution to other well known distributions, which will extend the value of the results.

Keywords: *Darboux theory; differential geometry; geodesic equation; Extreme Value distribution; moments of this distribution; second order partial differential equation; rotate axis; six related models; θ equal 22.8756235 4.*

Mathematical Subject Classification: 62E99

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1 Introduction

Extreme value distribution theory originated to assist astronomers in evaluating the validity of outlying observations. However, this distribution has also found a variety of applications related to natural phenomena such as rainfall, floods, wind gusts, air pollution and corrosion. The early papers by Fuller [1] and Griffith [2] on the subject were highly specialized; both in the fields of application and in the methods of mathematical analysis. This area of research thus attracted initially the interests of theoretical probabilists as well as engineers and hydrologists, and only recently of the mainstream statisticians. Historically work on extreme value length problems may be dated back to as early as 1709 when Nicolas Bernoulli discussed the mean largest distance from the origin when n points lie at random on a straight line of length t. (see Gumbel [3]) The most detailed bibliography that contains more than 350 references about this distribution can be found in Johnson, N.L., Kotz, S. and Balakrishnan N [4].

In this paper we focus on the geodesic equation aspect of the extreme value distribution theory. We use Darboux's theory to set up a second order partial differential equation. Later, we introduce a two variable chain rule method to rotate the main axis, *by 22.87562354 degree*, to remove the interaction terms. In this way, we can use the separable variable method to solve this second order partial differential equation. In section 5 we list and prove some related moments. Finally, we give six transformations which transform standard exponential distribution to some well known distributions that extend our results.

2 List the Fundamental Tensor

The standard form of the two-parameter extreme value distribution has the cumulative function and the probability density function given by,

$$F(x,u,v) = e^{-e^{-\frac{x-u}{v}}}; \tag{2.1}$$

$$f(x,u,v) = \frac{\partial F}{\partial x} = \frac{1}{v} e^{-e^{-\frac{x-u}{v}}} e^{-\frac{x-u}{v}} = \frac{1}{v}\exp(-e^{-\frac{x-u}{v}} - \frac{x-u}{v}); \tag{2.2}$$

$$\ln f(x) = -\ln v - e^{-\frac{x-u}{v}} - \frac{x-u}{v};$$

$$\text{where } u,v \text{ are parameters}, \quad -\infty < x < \infty,\ -\infty < u < \infty,\quad v > 0.$$

From the equation (2.1) and (2.2) above, we derive the basic metric tensor components for this distribution as follows,

$$E = -E\left(\frac{\partial^2 \ln f(x)}{\partial u^2}\right) = \frac{1}{v^2}, \tag{2.3}$$

$$F = -E\left(\frac{\partial^2 \ln f(x)}{\partial v \partial u}\right) = \frac{-(1-\gamma)}{v^2}, \tag{2.4}$$

$$G = -E(\frac{\partial^2 \ln f(x)}{\partial v^2}) = \frac{\Gamma^{(2)}(2)+1}{v^2}, \qquad (2.5)$$

$$EG - F^2 = \frac{1}{v^4}((1-\gamma)^2 + \frac{\pi^2}{6}) - \frac{(1-\gamma)^2}{v^4} = \frac{\pi^2}{6v^4}; \qquad (2.6)$$

Equation (2.3), (2.4), (2.5) and (2.6) will be used to set up the partial differential equation (3.1) in the next section.

$$\frac{\partial^2 \ln f(x)}{\partial v \partial u} = \frac{\partial^2 \ln f(x)}{\partial u \partial v} = \frac{-1}{v^2} + \frac{1}{v^2} e^{-\frac{x-u}{v}} - \frac{x-u}{v^3} e^{-\frac{x-u}{v}}; \qquad (2.7)$$

$$\frac{\partial^2 \ln f(x)}{\partial v^2} = \frac{1}{v^2} - [\frac{(x-u)^2}{v^4} e^{-\frac{x-u}{v}} - \frac{2(x-u)}{v^3} e^{-\frac{x-u}{v}}] - \frac{2(x-u)}{v^3}; \qquad (2.8)$$

Equation (2.7) and (2.8) used to define F and G.

In section IV, we will give more detail on how we find the expectation of these second partial derivatives.

3 The Geodesic Equation

In this section, we will find the geodesic equation of the extreme value distribution by solving one partial differential equation. This idea originated from Darboux's theory. Chen W. [5,6,7,8] has applied similar method in his previous paper to find the geodesic equation of inverse Gaussian and some other useful distribution. There are some other related useful references for example Kass RE, Vos PW [9], Struik DJ [10], and Grey A. [11]. To avoid confusion, we will only index those formulas that will use later and ignore the others. Base on result of section 2, we can easily set up $\nabla Z = 1$ as follows,

$$\frac{EZ_v^2 - 2FZ_uZ_v + GZ_u^2}{EG - F^2} = 1$$

$$\frac{1}{v^2} Z_v^2 + \frac{2(1-\gamma)}{v^2} Z_u Z_v + \frac{1}{v^2}((1-\gamma)^2 + \frac{\pi^2}{6}) Z_u^2 = \frac{\pi^2}{6v^4};$$

$$Z_v^2 + 2(1-\gamma) Z_u Z_v + ((1-\gamma)^2 + \frac{\pi^2}{6}) Z_u^2 = \frac{\pi^2}{6v^2}; \qquad (3.1)$$

where $\gamma = 0.57721566\ 49$ *known as Euler constant,*

$$(1-\gamma)^2 + \frac{\pi^2}{6} = 1.823680661$$

To solve the partial differential equation (3.1) above, we would consider the polar coordinate transformation. *Let* $u = r\cos\theta,\ v = r\sin\theta,$ We should keep in mind that Z is a function (u,v) while both (u,v) are also function of (r,θ). In calculus we learn that the chain rule will give us the following results.

$$Z_r = \frac{\partial Z}{\partial u}\frac{\partial u}{\partial r} + \frac{\partial Z}{\partial v}\frac{\partial v}{\partial r} = Z_u \cos\theta + Z_v \sin\theta,$$

$$Z_\theta = \frac{\partial Z}{\partial u}\frac{\partial u}{\partial \theta} + \frac{\partial Z}{\partial v}\frac{\partial v}{\partial \theta} = Z_u(-r\sin\theta) + Z_v r\cos\theta, \qquad (3.2)$$

Using Cramer Rule in equation (3.2), we can solve reversely of Z_u *and* Z_v as a function of Z_r *and* Z_θ as follows:

$$Z_u = \frac{\begin{vmatrix} Z_r & \sin\theta \\ Z_\theta & r\cos\theta \end{vmatrix}}{\begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix}} = \frac{r\cos\theta Z_r - \sin\theta Z_\theta}{r\cos^2\theta + r\sin^2\theta} = \cos\theta Z_r - \frac{1}{r}\sin\theta Z_\theta$$

$$Z_v = \frac{\begin{vmatrix} \cos\theta & Z_r \\ -r\sin\theta & Z_\theta \end{vmatrix}}{\begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix}} = \frac{\cos\theta Z_\theta + r\sin\theta Z_r}{r\cos^2\theta + r\sin^2\theta} = \sin\theta Z_r + \frac{1}{r}\cos\theta Z_\theta \qquad (3.3)$$

Substitute (3.3) into (3.1), we get

$$(\sin\theta\, Z_r + \frac{1}{r}\cos\theta\, Z_\theta)^2 + 2(1-\gamma)(\sin\theta\, Z_r + \frac{1}{r}\cos\theta\, Z_\theta)(\cos\theta\, Z_r - \frac{1}{r}\sin\theta\, Z_\theta)$$
$$+((1-\gamma)^2 + \frac{\pi^2}{6})(\cos\theta Z_r - \frac{1}{r}\sin\theta Z_\theta)^2 = \frac{\pi^2}{6v^2} \qquad (3.4)$$

Calculate the coefficient of $Z_r Z_\theta$

$$\frac{2}{r}\sin\theta\cos\theta + 2(1-\gamma)(-\frac{\sin^2\theta}{r} + \frac{\cos^2\theta}{r}) - ((1-\gamma)^2 + \frac{\pi^2}{6})\frac{2}{r}\sin\theta\cos\theta = 0$$
$$\sin 2\theta + 2(1-\gamma)\cos 2\theta - 1.823680661\sin 2\theta = 0$$
$$0.84556867 - 0.823680661\ \tan 2\theta = 0$$
$$\tan 2\theta = \frac{0.84556867}{0.823680661} = 1.026573416$$
$$2\theta = 45.75124707; \quad and \quad \theta = 22.87562354$$

Calculate the coefficient of Z_r^2

$$\sin^2\theta + 2(1-\gamma)\sin\theta\cos\theta + ((1-\gamma)^2 + \frac{\pi^2}{6})\cos^2\theta = 2.002059825$$

Calculate the coefficient of Z_θ^2

$$\frac{1}{r^2}\cos^2\theta + 2(1-\gamma)(-\frac{1}{r^2}\sin\theta\cos\theta) + ((1-\gamma)^2 + \frac{\pi^2}{6})\frac{\sin^2\theta}{r^2} = \frac{0.821620836}{r^2}$$

Constant term $\frac{\pi^2}{6v^2} = \frac{\pi^2}{6(r\sin\theta)^2} = \frac{10.88548832}{r^2}$

After rotation 22.87562354 then equation (3.4) becomes

$$2.002059825 Z_r^2 + \frac{0.821620836}{r^2} Z_\theta^2 = \frac{10.88548832}{r^2}; \text{ or}$$
$$2.002059825\, r^2 Z_r^2 = 10.88548832 - 0.821620836 Z_\theta^2 \tag{3.5}$$

We now can break above equation (3.5) into two separate parts and let them equal the same constant, say A^2,

$$2.002059825 r^2 Z_r^2 = A^2; \quad Z_r = \pm\frac{A}{\sqrt{2.002059825}\, r};$$

part 1,

$$Z = \pm\frac{A}{\sqrt{2.002059825}}\int\frac{dr}{r} = \pm\frac{A\ln r}{\sqrt{2.002059825}}; \tag{3.6}$$

$$10.88548832 - 0.821620836 Z_\theta^2 = A^2; \; Z_\theta = \pm\left(\frac{10.88548832 - A^2}{0.821620836}\right)^{\frac{1}{2}};$$

part 2,

$$Z = \pm\left(\frac{10.88548832 - A^2}{0.821620836}\right)^{\frac{1}{2}}\theta; \tag{3.7}$$

Put the equations (3.6) and (3.7) together and finally we arrived the general solution of equation (3.5),

$$Z = \pm\frac{A\ln r}{\sqrt{2.002059825}} \pm \left(\frac{10.88548832 - A^2}{0.821620836}\right)^{\frac{1}{2}}\theta;$$

Applying the Darboux Theory, we find that the geodesic equation of extreme value distribution is given by, $\frac{\partial Z}{\partial A} = B$;

$$\pm\frac{\ln r}{\sqrt{2.002059825}} \pm \frac{1}{2}\left(\frac{10.88548832 - A^2}{0.821620836}\right)^{-\frac{1}{2}}\frac{-2A}{0.821620836}\theta = B. \tag{3.8}$$

From previous defined relations, we know that (r,θ) and (u,v) are related to

$$r^2 = u^2 + v^2 \quad \text{and} \quad \tan\theta = \frac{\mathrm{v}}{\mathrm{u}};$$
$$or \;\; \mathrm{r} = \sqrt{u^2 + v^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{\mathrm{v}}{\mathrm{u}} \tag{3.9}$$

hence after substituting the relation (3.9) into equation (3.8) we find our geodesic equation of extreme value distribution as:

$$\pm\frac{ln(u^2+v^2)}{2\sqrt{2.002059825}}\pm\frac{1}{\sqrt{10.88548832-A^2}}\frac{Atan^{-1}\frac{v}{u}}{\sqrt{0.821620836}}=B$$

Where A, B are arbitrary constants.

4 Deriving the Basic Tensor

We demonstrated the detailed calculation of the four expected values that used to support the results in section 2. To simplify the complicated integrand we always use the following transformation.

$$\text{let } y = e^{-\frac{x-u}{v}};\ \ln y = -\frac{x-u}{v};\ x-u = -v\ln y;\ \text{or } x = -v\ln y + u;$$

$$\text{then } \frac{dx}{dy} = -\frac{v}{y};\quad dx = -\frac{v}{y}dy; \tag{4.1}$$

$$E(e^{-\frac{x-u}{v}}) = \frac{1}{v}\int_{-\infty}^{\infty} e^{-\frac{x-u}{v}} e^{-e^{-\frac{x-u}{v}}} e^{-\frac{x-u}{v}} dx = \int_0^{\infty} ye^{-y}dy = \Gamma(2) = 1. \tag{4.2}$$

$$E((x-u)e^{-\frac{x-u}{v}}) = \frac{1}{v}\int_{-\infty}^{\infty} (x-u)e^{-\frac{x-u}{v}} e^{-e^{-\frac{x-u}{v}}} e^{-\frac{x-u}{v}} dx$$

$$= -\int_0^{\infty} (v\ln y)ye^{-y}dy = -v\Gamma'(2) = -v(1-\gamma) \tag{4.3}$$

From (4.1) we substitute $y = e^{-\frac{x-u}{v}}$, $x = -v\ln y + u;$ and $dx = -\frac{v}{y}dy$ into (4.2) and (4.3) then integral turns out to be well known gamma and digamma function. In mathematical analysis by Apostol T. [12], p284, defined this ‘derivative of the gamma function’ as follows:

$$\Gamma'(t) = \int_0^{\infty} x^{t-1}\ln x\, e^{-x}dx.$$

This function obtained by differentiating the integral for $\Gamma(t)$ under the definite integral sign. The derivative $\Gamma'(t)$ exists for each t>0.

$$E((x-u)^2 e^{-\frac{x-u}{v}}) = \frac{1}{v}\int_{-\infty}^{\infty} (x-u)^2 e^{-\frac{x-u}{v}} e^{-e^{-\frac{x-u}{v}}} e^{-\frac{x-u}{v}} dx$$

$$= v^2\int_0^{\infty} (\ln y)^2 ye^{-y}dy = v^2\Gamma^{(2)}(2) = v^2(\psi'(2) + (\Gamma'(2)^2)$$

Apostol T. p303 defined the nth derivative of the gamma function as follows:

$$\Gamma^{(n)}(t)=\int_0^{\infty} x^{t-1}(\ln x)^n\, e^{-x}dx; \quad t>0$$

$$define\ \psi(x)=\frac{d}{dx}\ln\Gamma(x)=\frac{\Gamma'(x)}{\Gamma(x)}=\sum_{k=0}^{\infty}\left(\frac{1}{k+1}-\frac{1}{x+k}\right)-\gamma,$$

$$\psi(1)=\Gamma'(1)=-\gamma.$$

$$\psi(2)=\frac{\Gamma'(2)}{\Gamma(2)}=\Gamma'(2)=\sum_{k=0}^{\infty}\left(\frac{1}{k+1}-\frac{1}{2+k}\right)-\gamma=1-\gamma,$$

$$\psi(2)=\Gamma'(2)=1-\gamma=1-0.5772156649=0.422784335$$

$$\psi'(x)=\frac{\Gamma(x)\Gamma^{(2)}(x)-(\Gamma'(x))^2}{(\Gamma(x))^2}$$

$$\psi'(2)=\Gamma^{(2)}(2)-(\Gamma'(2))^2$$

$$\Gamma^{(2)}(2)=\psi'(2)+(\Gamma'(2))^2=(1-\gamma)^2+\frac{\pi^2}{6}-1.$$

$$\psi'(2)=\psi'(1)-1,\ and\ \psi'(1)=\sum_{k=0}^{\infty}\frac{1}{(k+1)^2}=\frac{\pi^2}{6}.$$

5 List and Prove Some Related Moments

(5.1) mean: $u+\gamma v \quad where \quad \gamma\ is\ Euler's\ constant$

(5.2) variance: $\frac{\pi^2 v^2}{6}$

(5.3) mode: u

(5.4) median: $u-v\ \ln(\ln 2)$

(5.5) coefficient of variation: $\frac{\pi v}{(u+v\gamma)\sqrt{6}}$

Assume random variable x has extreme value distribution then we can find the moment generating function of x:

$$M_x(t)=E(e^{tx})=\int_{-\infty}^{\infty} e^{tx}\frac{1}{v}e^{-e^{-\frac{x-u}{v}}}e^{-\frac{x-u}{v}}dx$$

$$=\int_0^{\infty} y^{1-vt-1}e^{tu}e^{-y}dy=e^{tu}\Gamma(1-vt) \qquad\qquad vt<1.$$

Then as usual we can take derivative of $M_x(t)$ and let t=0 find the mean value of the distribution.

$$M'_x(t) = e^{tu}(u\Gamma(1-vt) - v\Gamma'(1-vt));$$
$$M'_x(0) = E(x) = u\Gamma(1) - v\Gamma'(1) = u - v(-\gamma)$$
$$(5.1)\ \ mean:\ u + v\gamma\ \ where\ -\gamma = \Gamma'(1);\ \gamma \text{ is Euler constant.}$$
$$M''_x(t) = e^{tu}(u\Gamma'(1-vt)(-v) - v\Gamma''(1-vt)(-v)) +$$
$$e^{tu}u(u\Gamma(1-vt) - v\Gamma'(1-vt))$$
$$M''_x(0) = E(x^2) = (-uv\Gamma'(1) + v^2\Gamma''(1)) + u(u - v\Gamma'(1))$$
$$(5.2)\ Var(x) = E(x^2) - (E(x))^2$$
$$= (-uv\Gamma'(1) + v^2\Gamma''(1)) + u^2 - uv\Gamma'(1)) - (u - v\Gamma'(1))^2$$
$$= v^2(\Gamma''(1) - (\Gamma'(1)^2) = v^2\Psi'(1) = \frac{\pi^2 v^2}{6}$$
$$\ln f(x) = -\ln v - e^{-\frac{x-u}{v}} - \frac{x-u}{v} \qquad e^{-\frac{x-u}{v}} = 1;\ \ -\frac{x-u}{v} = \ln 1 = 0$$
$$\frac{\partial \ln f(x)}{\partial x} = -e^{-\frac{x-u}{v}}(-\frac{1}{v}) - \frac{1}{v} = 0; \qquad (5.3)\ \text{hence}\ \ x = u \text{ is the mode.}$$
$$F(x,u,v) = e^{-e^{-\frac{x-u}{v}}} = \frac{1}{2}; \qquad -\infty < x < \infty,\ -\infty < u < \infty,\ \ v > 0.$$
$$-e^{-\frac{x-u}{v}} = -\ln 2,\ \ -\frac{x-u}{v} = \ln(\ln 2);$$
$$(5.4)\ \ x = u - v\ln(\ln 2)\ \ \text{is the median.}$$
$$(5.5)\ \ \text{it is trivial that cv is } \frac{\pi v}{(u+v\gamma)\sqrt{6}}.\ \text{Simply from definition.}$$

6 Conclusion and Remarks

There are three types of extreme value distribution for maxima and three corresponding types of extreme value distribution for minima. The term extreme value distributions includes all distributions with cumulative distribution function:

$$G_k(\frac{x-u}{v})\ \ and\ \ H_k(\frac{x-u}{v})\ ,\quad k = 1,2,3$$

with the standard member when U=0 and V=1 (see appendix.) The name extreme value distribution has been used in literature only for distributions with a cumulative distribution $H_3(\frac{x-u}{v})$. We should keep in mind that all six types of extreme value distributions given in the appendix are closely related to exponential distributions. Let x have a standard exponential distribution and the six transformed random variables have

$$-X^{-\frac{1}{\alpha}}, \; X^{\frac{1}{\alpha}}, \; \ln X, \quad X^{-\frac{1}{\alpha}}, \; -X^{\frac{1}{\alpha}}, \quad -\ln X \tag{6.1}$$

respectively, the distributions

$$G_{1,\alpha}\;, G_{2,\alpha}\,, \; G_3\,, \;\; H_{1,\alpha,} \quad H_{2,\alpha}\,, \quad H_3$$

Using the method suggested by Balakrishnan N. and Nevzorov V.B. [13] on page 194~196, we can easily find the other six related distribution corresponding moments from (6.1). In this way it may fit more applications.

Competing Interests

Author has declared that no competing interests exist.

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APPENDIX

We list the six corresponding cumulative distribution function as follows;

$$G_1(x) = G_{1,\alpha}(x) = \begin{cases} 1 - e^{-(-x)^{-\alpha}} & x < 0,\ \alpha > 0. \\ 1 & x \geq 0. \end{cases},$$

$$G_2(x) = G_{2,\alpha}(x) = \begin{cases} 0 & x < 0, \\ 1 - e^{-x^{\alpha}} & x > 0,\ \alpha > 0. \end{cases},$$

$$G_3(x) = 1 - e^{-e^{x}}, \qquad -\infty < x < \infty$$

All cumulative distribution functions

$$G_k(\frac{x-u}{v}), \qquad k = 1,2,3,$$
$$\text{where } -\infty < u < \infty, \text{and } v > 0,$$

can also be limiting distribution of minimal values.

G_2 is commonly known as the Weibull distribution.

$$H_1(x) = H_{1,\alpha}(x) = \begin{cases} 0 & x < 0, \\ e^{-x^{-\alpha}} & x > 0,\ \alpha > 0. \end{cases},$$

$$H_2(x) = H_{2,\alpha}(x) = \begin{cases} e^{-(-x)^{\alpha}} & x < 0,\ \alpha > 0 \\ 1 & x > 0; \end{cases},$$

$$H_3(x) = e^{-e^{-x}}, \qquad -\infty < x < \infty$$

All cumulative distribution functions

$$H_k(\frac{x-u}{v}), \qquad k = 1,2,3,$$
$$\text{where } -\infty < u < \infty, \text{and } v > 0,$$

can also be limiting distribution of normalized maxima values. H_1 is called the Frechet-type distribution, H_2 the Weibull-type distribution, and H_3 is also referred to in the literature as the log-weibull, double exponential, and doubly exponential distribution.

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