Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 2412012, 15 pages
https://doi.org/10.1155/2022/2412012

Research Article

A Novel Description of Some Concepts in Interval-Valued Intuitionistic Fuzzy Graph with an Application

Xiaoli Qiang,[1] Saeed Kosari ,[1] Xiang Chen,[1] Ali Asghar Talebi,[2] Ghulam Muhiuddin ,[3] and Seyed Hossein Sadati[2]

[1]*Institute of Computing Science and Technology, Guangzhou University, Guangzhou 510006, China*
[2]*Department of Mathematics, University of Mazandaran, Babolsar, Iran*
[3]*Department of Mathematics, University of Tabuk, Tabuk 71491, Saudi Arabia*

Correspondence should be addressed to Saeed Kosari; saeedkosari38@gzhu.edu.cn

Received 22 April 2022; Revised 7 June 2022; Accepted 21 June 2022; Published 8 July 2022

Academic Editor: Ranjan Kumar

Covering, matching, and domination are the basic concepts in graphs that play a decisive role in the properties of graphs. Calculating these parameters is one of the difficulties in fuzzy graphs when it is not possible to accurately determine the values of the vertices of a graph. The interval-valued intuitionistic fuzzy graph (IVIFG) is one of the fuzzy graphs which can play an important role in solving uncertain problems in different sciences such as psychology, biological sciences, medicine, and social networks. The necessity of using a range of value instead of one number caused them to help researchers in optimizing and saving time and cost. In this study, we introduce some of the specific concepts such as covering, matching, and paired domination using strong arc or effective edges by giving appropriate examples. In addition, we have calculated strong node covering number, strong independent number, and other parameters of complete bipartite IVIFGs with several examples. Finally, we have presented an application of IVIFG in social networks.

1. Introduction

Graphs are an inevitable tool in applied mathematics. Among the various concepts in graphs, some concepts are more important such as covering, matching, and domination. These concepts are closely related to vertices, as one of the most important components of the graph, and cause them to participate in many analyses related to vertices. Many studies have been done by researchers on various graphs. It is difficult to examine these concepts when the exact values for the vertices cannot be considered.

In 1965, Zadeh [1] presented the basic idea of fuzzy set (FS) where its prominent feature was the allocation of membership degree between 0 and 1 to each element in a set. Zadeh [2] also introduced the interval-valued fuzzy set (IVFS) in 1975, in which membership degrees were intervals of numbers. Roselfeld [3] defined a new concept called the fuzzy graph (FG) by employing fuzzy relations on FS. FGs were considered by researchers in the fields related to ambiguous and uncertain problems. They were able to find numerous applications in solving and modeling problems in computer science, engineering, system analysis, economics, network routing, transportation, and so on. With the advent of new indefinite problems, it became clear that a membership function could not well express the ambiguity in subjective perceptions and the complexity of data. To overcome this shortcoming of the FS, Atanassov [4] proposed an extension of FS by introducing nonmembership function and defined intuitionistic fuzzy set (IFS). IFGs were first introduced by Atanassov [5] in 1999 and was further discussed in [6]. Mahapatra et al. [7–9] explored concepts on fuzzy graphs. Rashmanlou and Pal [10, 11] studied different kinds of FGs. Kosari et al. [12] presented vague graph structure with an application in medical diagnosis. Kou et al. [13] studied some properties of vague graph (VG). Krishna et al. [14] studied new results in cubic graphs. Talebi et al. [15, 16] defined Cayley-FGs and some operations on level graphs of bipolar FGs. Atanassov [17] recently introduced some new topological operators over IFSs. Mathew et al. [18] conducted research on vertex rough graphs. Some concepts in IVFGs and neutrosophic graphs are studied by Jan et al. [19]. Voskoglou [20] used a combination of soft sets and gray numbers in decision-making. Mahapatra et al. [21–23] introduced concepts of neutrosophic graphs used in social networks.

The decision to determine accurate numerical values in uncertain and inaccurate evaluations of information, which often occur in practical situations, is associated with difficulties. Thus, in 1989, Atanassov and Gargov [24] introduced the idea of the interval-valued intuitionistic fuzzy set (IVIFS) in order to unify perceptions and quantify the uncertain nature of the mind. This concept is defined by a membership function, a nonmembership function, and a hesitant function whose values are intervals between 0 and 1 instead of exact numbers. IVIFS has been widely used in many areas, such as decision-making [25], pattern recognition [26], medical diagnosis [27], and graph theory [28]. The concept of interval-valued fuzzy graphs (IVFGs) is presented by Hongmei and Lianhua in [29]. Akram et al. [30, 31] defined certain types of IVFGs. The product of IVIFGs was proposed by Mishra and Pal in [32]. The strong IVIFG concept is described by Ismayil and Ali [33]. Rashmanlou et al. [25, 34–36] studied some IVIFG concepts.

The purpose of this paper is to find a way to determine the concepts of vertex covering, matching, and paired domination in IVIFGs where we are dealing with interval-valued numbers instead of fuzzy numbers. The previous definition limitations in the vertex covering, matching, and paired domination of FGs have directed us to offer new classifications in terms of IVIFG. These concepts have already been studied by some researchers in a variety of FGs. Sahoo et al. [37] investigated covering and paired domination in IFGs.

The rest of this article is organized as follows: Section 2 briefly reviews related basic concepts to IVIFGs. In Section 3, we introduced the concepts of strong vertex covering, independent vertex covering, and perfect strong matching in an IVIFG by strong edges and defined some of its properties in specific types of IVIFGs. In this section, we introduce paired domination in IVIFG and examine its implications. Finally, we present an application of IVIFG on social networks in Section 4.

2. Preliminaries

In this section, we briefly define some of the basic concepts for entering the main discussion.

Definition 1 (see [24]). An IVIFS A in X can be described as

$$A = \left\{ \left\langle x, \left[\mu_A^L(x), \mu_A^U(x)\right], \left[\nu_A^L(x), \nu_A^U(x)\right] \right\rangle | x \in X \right\}, \quad (1)$$

where $0 \le \mu_A^L(x) \le \mu_A^U \le 1$, $0 \le \nu_A^L(x) \le \nu_A^U \le 1$, and $0 \le \mu_A^U(x) + \nu_A^U(x) \le 1$, for all $x \in X$.

Similarly, the intervals $[\mu_A^L(x), \mu_A^U(x)]$ and $[\nu_A^L(x), \nu_A^U(x)]$ denoted the MV and non-MV of an element x, respectively. If each of the intervals contains only one value for each $x \in X$, we have

$$\mu_A(x) = \mu_A^L(x) = \mu_A^U(x), \nu_A(x) = \nu_A^L(x) = \nu_A^U(x). \quad (2)$$

Furthermore, the hesitancy degree of each element x is as follows:

$$\left[1 - \mu_A^L(x) - \nu_A^L(x), 1 - \mu_A^U(x) - \nu_A^U(x)\right]. \quad (3)$$

Definition 2 (see [33]). An IVIFG of an underlying graph $G^* = (V, E)$ is a pair $G = (V, A, B)$ so that

$$A = \left\{ \left\langle \left[\mu_A^L(x), \mu_A^U(x)\right], \left[\nu_A^L(x), \nu_A^U(x)\right] \right\rangle | x \in V \right\}, \quad (4)$$

is an IVIFS in V and

$$B = \left\{ \left\langle \left[\mu_B^L(xy), \mu_B^U(xy)\right], \left[\nu_B^L(xy), \nu_B^U(xy)\right] \right\rangle | xy \in E \right\}, \quad (5)$$

is an interval-valued intuitionistic fuzzy relation (IVIFR) $V \times V$ so that

$$\begin{aligned}
\mu_B &: E \subseteq V \times V \longrightarrow D[0, 1], \\
\nu_B &: E \subseteq V \times V \longrightarrow D[0, 1], \\
\mu_B^L(xy) &\le \min \left\{\mu_A^L(x), \mu_A^L(y)\right\}, \\
\mu_B^U(xy) &\le \min \left\{\mu_A^U(x), \mu_A^U(y)\right\}, \\
\nu_B^L(xy) &\ge \max \left\{\nu_A^L(x), \nu_A^L(y)\right\}, \\
\nu_B^U(xy) &\ge \max \left\{\nu_A^U(x), \nu_A^U(y)\right\},
\end{aligned} \quad (6)$$

and $\mu_B^U(xy) + \nu_B^U(xy) \le 1$, for each $xy \in E$.

Definition 3 (see [34]). An edge xy of an IVIFG, G is named a strong arc (SA) or effective edge if

$$\begin{aligned}
\mu_B^L(xy) &= \min \left\{\mu_A^L(x), \mu_A^L(y)\right\}, \\
\mu_B^U(xy) &= \min \left\{\mu_A^U(x), \mu_A^U(y)\right\}, \\
\nu_B^L(xy) &= \max \left\{\nu_A^L(x), \nu_A^L(y)\right\}, \\
\nu_B^U(xy) &= \max \left\{\nu_A^U(x), \nu_A^U(y)\right\}.
\end{aligned} \quad (7)$$

Definition 4 (see [34]). An IVIFG is complete, if

$$\begin{aligned}
\mu_B^L(xy) &= \min \left\{\mu_A^L(x), \mu_A^L(y)\right\}, \\
\nu_B^L(xy) &= \max \left\{\nu_A^L(x), \nu_A^L(y)\right\}, \\
\mu_B^U(xy) &= \min \left\{\mu_A^U(x), \mu_A^U(y)\right\}, \\
\nu_B^U(xy) &= \max \left\{\nu_A^U(x), \nu_A^U(y)\right\},
\end{aligned} \quad (8)$$

for all $xy \in V \times V$.

As a result of the above definition, the following definition can be provided.

Definition 5. An IVIFG G is named bipartite whenever the vertex set V can be partitioned into two nonempty sets V_1

and V_2 so that $\mu_B^L(xy)=\mu_B^U(xy)=0$ and $\nu_B^L(xy)=\nu_B^U(xy)=0$, for $xy\in V_1$ or $xy\in V_2$. If

$$\begin{aligned}\mu_B^L(xy)&=\min\left\{\mu_A^L(x),\mu_A^L(y)\right\},\\ \mu_B^U(xy)&=\min\left\{\mu_A^U(x),\mu_A^U(y)\right\},\\ \nu_B^L(xy)&=\max\left\{\nu_A^L(x),\nu_A^L(y)\right\},\\ \nu_B^U(xy)&=\max\left\{\nu_A^U(x),\nu_A^U(y)\right\},\end{aligned}\tag{9}$$

for all $x\in V_1$ and $y\in V_2$; then, G is named a complete bipartite IVIFG (CB-IVIFG) and is shown by K_{σ_1,σ_2}.

All the basic notations are shown in Table 1.

3. Covering, Matching, and Paired Domination in the IVIFGs

In this section, we introduce covering, matching, and paired domination in the IVIFGs by the weight of strong edges and examine some of its properties and results.

Definition 6. Let $G=(V,A,B)$ be an IVIFG. An SNC in an IVIFG G is the set D of nodes that cover all SAs of G. The weight of an SNC D is denoted as

$$\begin{aligned}W_{nc}&=\left\langle\left[W_{nc}^{L_\mu}(D),W_{nc}^{U_\mu}(D)\right],\left[W_{nc}^{L_\nu}(D),W_{nc}^{U_\nu}(D)\right]\right\rangle,\\ W_{nc}&=\left\langle\left[\sum_{x\in D}\mu_B^L(xy),\sum_{x\in D}\mu_B^U(xy)\right],\left[\sum_{x\in D}\nu_B^L(xy),\sum_{x\in D}\nu_B^U(xy)\right]\right\rangle,\end{aligned}\tag{10}$$

so that $\mu_B^L(xy)$ and $\mu_B^U(xy)$ are the minimum of the lower and upper of IVMBs and $\nu_B^L(xy)$ and $\nu_B^U(xy)$ are the maximum of the lower and upper of IVNMBs of all SAs incident on x, respectively.

An SNCN of an IVIFG G is shown as follows $\alpha_{s_0}(G)=\alpha_{s_0}=\langle[\alpha_{s_0}^{L_\mu},\alpha_{s_0}^{U_\mu}],[\alpha_{s_0}^{L_\nu},\alpha_{s_0}^{U_\nu}]\rangle$ so that

$$\begin{aligned}\alpha_{s_0}^{L_\mu}&=\min\left\{W_{nc}^{L_\mu}\middle|D\text{ is the weight of SNCs of }G\right\},\\ \alpha_{s_0}^{U_\mu}&=\min\left\{W_{nc}^{U_\mu}\middle|D\text{ is the weight of SNCs of }G\right\},\\ \alpha_{s_0}^{L_\nu}&=\max\left\{W_{nc}^{L_\nu}\middle|D\text{ is the weight of SNCs of }G\right\},\\ \alpha_{s_0}^{U_\nu}&=\max\left\{W_{nc}^{U_\nu}\middle|D\text{ is the weight of SNCs of }G\right\}.\end{aligned}\tag{11}$$

A minimum SNC in an IVIFG G is an SNC of minimum IVMBs and maximum IVNMBs.

Table 1: Some basic notations.

Notation	Meaning
IVFS	Interval-valued fuzzy set
IFG	Intuitionistic fuzzy graph
IVIFS	Interval-valued intuitionistic fuzzy set
IVIFG	Interval-valued intuitionistic fuzzy graph
CIVIFG	Complete interval-valued intuitionistic fuzzy graph
SA	Strong arc
CB	Complete bipartite
SC	Strong cover
SCN	Strong covering number
SNC	Strong node cover
ISA	Incident strong arc
SNCN	Strong node covering number
IVMB	Interval-valued membership bound
IVNMB	Interval-valued nonmembership bound
SI	Strong independent
SIS	Strong independent set
SIN	Strong independent number
IN	Isolated node
SAC	Strong arc cover
PD	Paired domination
SPDN	Strong paired domination number
SM	Strong matching
SMN	Strong matching number
SIAC	Strong independent arc cover
PSM	Perfect strong matching
SDS	Strong dominating set
SPDS	Strong paired dominating set

Theorem 7. *Let $G=(V,A,B)$ be a CIVIFG. Then,*

$$\begin{aligned}\alpha_{s_0}^{L_\mu}&=(r-1)\mu_B^L(xy),\\ \alpha_{s_0}^{L_\nu}&=(r-1)\nu_B^L(xy),\\ \alpha_{s_0}^{U_\mu}&=(r-1)\mu_B^U(xy),\\ \alpha_{s_0}^{U_\nu}&=(r-1)\nu_B^U(xy),\end{aligned}\tag{12}$$

where $\mu_B^L(xy)$ and $\mu_B^U(xy)$ are the lower and upper of IVMBs and $\nu_B^L(xy)$ and $\nu_B^U(xy)$ are the lower and upper of IVNMBs of the weakest arc in G. Note that r is the number of vertex in G.

Proof. Since G is a CIVIFG, all arcs are strong, and every node is neighbor to all other vertices. So, any set includes $(r-1)$ nodes forming an SNC of G.

Let x be a vertex having minimum of IVMBs and maximum of IVNMBs in G. Suppose $y_1,y_2\cdots,y_{n-1}$ is the node neighbor to x. Then, the $(r-1)$ arcs $xy_1,xy_2,\cdots,xy_{r-1}$ are all weakest arcs of G, and strength of each arcs is equal to $\langle[\mu_B^L(xy),\mu_B^U(xy)],[\nu_B^L(xy),\nu_B^U(xy)]\rangle$, which $y\in\{y_1,y_2,\cdots,y_{r-1}\}$.

Hence, the set $D=\{y_1, y_2,\cdots,y_{r-1}\}$ of $(r-1)$ vertices forms an SNC of G with

$$W_{nc}^{L_\mu}(D)=\sum_{y_i\in D}\mu_B^L(xy_i)=\mu_B^L(xy_1)+\mu_B^L(xy_2)+\cdots+\mu_B^L(xy_{r-1}),$$
$$W_{nc}^{U_\mu}(D)=\sum_{y_i\in D}\mu_B^U(xy_i)=\mu_B^U(xy_1)+\mu_B^U(xy_2)+\cdots+\mu_B^U(xy_{r-1}),\tag{13}$$

where $\mu_B^L(xy_i), i=1,2,\cdots,(r-1)$ is the minimum lower of IVMB and $\mu_B^U(xy_i), i=1,2,\cdots,(r-1)$ is the minimum upper of IVNMB of SAs incident on y_i. Then,

$$\alpha_{s_0}^{L_\mu}=\mu_B^L(xy)+\mu_B^L(xy)+\cdots+\mu_B^L(xy),$$
$$\alpha_{s_0}^{U_\mu}=\mu_B^U(xy)+\mu_B^U(xy)+\cdots+\mu_B^U(xy),\tag{14}$$

where $\mu_B^L(xy)$ and $\mu_B^U(xy)$ are the lower and upper of IVMBs of a weakest arc in G.

Hence, $\alpha_{s_0}^{L_\mu}=(r-1)\mu_B^L(xy)$ and $\alpha_{s_0}^{U_\mu}=(r-1)\mu_B^U(xy)$. Similarly,

$$W_{nc}^{L_\nu}(D)=\sum_{y_i\in D}\nu_B^L(xy_i)=\nu_B^L(xy_1)+\nu_B^L(xy_2)+\cdots+\nu_B^L(xy_{r-1}),$$
$$W_{nc}^{U_\nu}(D)=\sum_{y_i\in D}\nu_B^U(xy_i)=\nu_B^U(xy_1)+\nu_B^U(xy_2)+\cdots+\nu_B^U(xy_{r-1}),\tag{15}$$

where $\nu_B^L(xy_i)$ and $\nu_B^U(xy_i)$, $i=1,2,\cdots,(r-1)$ are the maximum lower and upper of IVNMBs of all SAs incident on y_i. Then,

$$\alpha_{s_0}^{L_\nu}=\nu_B^L(xy)+\nu_B^L(xy)+\cdots+\nu_B^L(xy),$$
$$\alpha_{s_0}^{U_\nu}=\nu_B^U(xy)+\nu_B^U(xy)+\cdots+\nu_B^U(xy),\tag{16}$$

where $\nu_B^L(xy)$ and $\nu_B^U(xy)$ are the lower and upper of IVNMBs of a weakest arc in G. Hence, $\alpha_{s_0}^{L_\nu}=(r-1)\nu_B^L(xy)$ and $\alpha_{s_0}^{U_\nu}=(r-1)\nu_B^U(xy)$. □

Theorem 8. *For a CB-IVIFG K_{σ_1,σ_2} with partite set V_1 and V_2,*

$$\alpha_{s_0}^{L_\mu}(K_{\sigma_1,\sigma_2})=\min\left\{W_{nc}^{L_\mu}(V_1), W_{nc}^{L_\mu}(V_2)\right\},$$
$$\alpha_{s_0}^{U_\mu}(K_{\sigma_1,\sigma_2})=\min\left\{W_{nc}^{U_\mu}(V_1), W_{nc}^{U_\mu}(V_2)\right\},$$
$$\alpha_{s_0}^{L_\nu}(K_{\sigma_1,\sigma_2})=\max\left\{W_{nc}^{L_\nu}(V_1), W_{nc}^{L_\nu}(V_2)\right\},$$
$$\alpha_{s_0}^{U_\nu}(K_{\sigma_1,\sigma_2})=\max\left\{W_{nc}^{U_\nu}(V_1), W_{nc}^{U_\nu}(V_2)\right\}.\tag{17}$$

Proof. All arcs in K_{σ_1,σ_2} are strong, and each node in V_1 is neighbor with all nodes in V_2 and contrariwise. The set of all arcs of K_{σ_1,σ_2} is a set of all arcs incident on each node of V_1 or a set of all arcs incident on each node of V_2. Hence, all SNCs in K_{σ_1,σ_2} are V_1, V_2, and $V_1\cup V_2$. Clearly, $W_{nc}^{L_\mu}(V_1\cup V_2)$ is greater than $W_{nc}^{L_\mu}(V_1)$ and $W_{nc}^{L_\mu}(V_2)$. Hence,

$$\alpha_{s_0}^{L_\mu}(K_{\sigma_1,\sigma_2})=\min\left\{W_{nc}^{L_\mu}(V_1), W_{nc}^{L_\mu}(V_2)\right\}.\tag{18}$$

Similarly, $\alpha_{s_0}^{U_\mu}(K_{\sigma_1,\sigma_2})=\min\{W_{nc}^{U_\mu}(V_1), W_{nc}^{U_\mu}(V_2)\}$. Also, $W_{nc}^{L_\nu}(V_1\cup V_2)$ is less than $W_{nc}^{L_\nu}(V_1)$ and $W_{nc}^{L_\nu}(V_2)$. So,

$$\alpha_{s_0}^{L_\nu}(K_{\sigma_1,\sigma_2})=\max\left\{W_{nc}^{L_\nu}(V_1), W_{nc}^{L_\nu}(V_2)\right\}.\tag{19}$$

In the same way, we have $\alpha_{s_0}^{U_\nu}(K_{\sigma_1,\sigma_2})=\max\{W_{nc}^{U_\nu}(V_1), W_{nc}^{U_\nu}(V_2)\}$. □

Definition 9. In an IVIFG G, two nodes are said to be SI if there is no SA between them. A set of nodes in G is an SI if and only if two nodes are in an SI set.

Definition 10. The weight of an SIS D in an IVIFG G is described as

$$W_{\text{is}}(D)=\left\langle\left[W_{\text{is}}^{L_\mu}(D), W_{\text{is}}^{U_\mu}(D)\right],\left[W_{\text{is}}^{L_\nu}(D), W_{\text{is}}^{U_\nu}(D)\right]\right\rangle,\tag{20}$$

i.e.,

$$W_{\text{is}}(D)=\left\langle\left[\sum_{x\in D}\mu_B^L(xy), \sum_{x\in D}\mu_B^U(xy)\right],\left[\sum_{x\in D}\nu_B^L(xy), \sum_{x\in D}\nu_B^U(xy)\right]\right\rangle,\tag{21}$$

where $\mu_B^L(xy)$ and $\mu_B^U(xy)$ are minimum of the lower and upper of IVMBs and $\nu_B^L(xy)$ and $\nu_B^U(xy)$ are maximum of the lower and upper of IVNMBs of all SAs incident on x, respectively.

An SIN of an IVIFG G is shown by $\beta_{s_0}(G)=\beta_{s_0}=\langle[\beta_{s_0}^{L_\mu}, \beta_{s_0}^{U_\mu}], [\beta_{s_0}^{L_\nu}, \beta_{s_0}^{U_\nu}]\rangle$, which

$$\beta_{s_0}^{L_\mu}=\max\left\{W_{\text{is}}^{L_\mu}(D)\,\middle|\,D\text{ is the SISs of nodes in }G\right\},$$
$$\beta_{s_0}^{U_\mu}=\max\left\{W_{\text{is}}^{U_\mu}(D)\,\middle|\,D\text{ is the SISs of nodes in }G\right\},$$
$$\beta_{s_0}^{L_\nu}=\min\left\{W_{\text{is}}^{L_\nu}(D)\,\middle|\,D\text{ is the SISs of nodes in }G\right\},$$
$$\beta_{s_0}^{U_\nu}=\min\left\{W_{\text{is}}^{U_\nu}(D)\,\middle|\,D\text{ is the SISs of nodes in }G\right\}.\tag{22}$$

A maximum SIS in an IVIFG G is an SIS with the maximum IVMBs and minimum IVNMBs.

Theorem 11. *Let G be a CIVIFG. Then,*

$$\beta_{s_0}(G)=\left\langle\left[\mu_B^L(xy),\mu_B^U(xy)\right],\left[\nu_B^L(xy),\nu_B^U(xy)\right]\right\rangle, \tag{23}$$

where $\mu_B^L(xy)$ and $\mu_B^U(xy)$ are the lower and upper of IVMBs and $\nu_B^L(xy)$ and $\nu_B^U(xy)$ are the lower and upper of IVNMBs of a weakest arc in G.

Proof. Since G is a CIVIFG, so all arcs are strong, and also, each arc is neighbor to all other nodes. Hence, $D=\{x\}$ is the only SIS for each $x\in V$. Thus, the result is true. □

Theorem 12. *Let K_{σ_1,σ_2} be a CB-IVIFG with partite set V_1 and V_2. Then,*

$$\begin{aligned}\beta_{s_0}^{L_\mu}(K_{\sigma_1,\sigma_2})&=\max\left\{W_{is}^{L_\mu}(V_1),W_{is}^{L_\mu}(V_2)\right\},\\ \beta_{s_0}^{U_\mu}(K_{\sigma_1,\sigma_2})&=\max\left\{W_{is}^{U_\mu}(V_1),W_{is}^{U_\mu}(V_2)\right\},\\ \beta_{s_0}^{L_\nu}(K_{\sigma_1,\sigma_2})&=\min\left\{W_{is}^{L_\nu}(V_1),W_{is}^{L_\nu}(V_2)\right\},\\ \beta_{s_0}^{U_\nu}(K_{\sigma_1,\sigma_2})&=\min\left\{W_{is}^{U_\nu}(V_1),W_{is}^{U_\nu}(V_2)\right\}.\end{aligned} \tag{24}$$

Proof. In K_{σ_1,σ_2} all arcs are strong. Also, each node in V_1 is neighbor with all nodes in V_2 and contrariwise. Therefore, all SISs in K_{σ_1,σ_2} are V_1 and V_2. Hence, the result is true. □

Example 1. Consider an IVIFG G is drawn in Figure 1.

Clearly, all arcs are strong, and all SNCs of G are as follows:

$$\begin{aligned}D_1&=\{y,t\},\\ D_2&=\{x,y,z\},\\ D_3&=\{x,z,t\},\\ D_4&=\{y,z,t\},\\ D_5&=\{x,y,t\},\\ D_6&=\{x,y,z,t\}.\end{aligned} \tag{25}$$

Table 2 shows the method of calculating the weight of SISs.

Thus, $\alpha_{s_0}=\langle[0.2,0.4],[0.8,2]\rangle$.

Example 2. Consider a strong IVIFG G is drawn in Figure 2. All SISs in G are $D_1=\{x,z\}, D_2=\{y,t\}$. The calculation of the weight of SIS is shown in Table 3. Therefore, $\beta_{s_0}=\langle[0.4,1],[0.7,0.9]\rangle$.

Definition 13. Let G be an IVIFG without INs. The weight of an SAC Y is described as $W_{ac}(Y)=\langle[W_{ac}^{L_\mu}(Y),W_{ac}^{U_\mu}(Y)],[W_{ac}^{L_\nu}(Y),W_{ac}^{U_\nu}(Y)]\rangle$, which $W_{ac}(Y)=\langle[\sum_{xy\in Y}\mu_B^L(xy),\sum_{xy\in Y}\mu_B^U(xy)],[\sum_{xy\in Y}\nu_B^L(xy),\sum_{xy\in Y}\nu_B^U(xy)]\rangle$.

An SACN of an IVIFG G is denoted by $\alpha_{s_1}(G)=\alpha_{s_1}=\langle[\alpha_{s_1}^{L_\mu},\alpha_{s_1}^{U_\mu}],[\alpha_{s_1}^{L_\nu},\alpha_{s_1}^{U_\nu}]\rangle$, where

$$\begin{aligned}\alpha_{s_1}^{L_\mu}&=\min\left\{W_{ac}^{L_\mu}(Y)\,\middle|\,Y\text{ is the SACs of }G\right\},\\ \alpha_{s_1}^{U_\mu}&=\min\left\{W_{ac}^{U_\mu}(Y)\,\middle|\,Y\text{ is the SACs of }G\right\},\\ \alpha_{s_1}^{L_\nu}&=\max\left\{W_{ac}^{L_\nu}(Y)\,|\,Y\text{ is the SACs of }G\right\},\\ \alpha_{s_1}^{U_\nu}&=\max\left\{W_{ac}^{U_\nu}(Y)\,|\,Y\text{ is the SACs of }G\right\}.\end{aligned} \tag{26}$$

A minimum SAC in an IVIFG G is an SAC with minimum IVMBs and maximum IVNMBs.

Theorem 14. *If G is a complete IVIFG, then*

$$\begin{aligned}\alpha_{s_1}^{L_\mu}&=\min\left\{W_{ac}^{L_\mu}(Y)\,\middle|\,Y\textit{ is a SAC in }G\textit{ with }|Y|\geq\left\lceil\frac{n}{2}\right\rceil\right\},\\ \alpha_{s_1}^{U_\mu}&=\min\left\{W_{ac}^{U_\mu}(Y)\,\middle|\,Y\textit{ is a SAC in }G\textit{ with }|Y|\geq\left\lceil\frac{n}{2}\right\rceil\right\},\\ \alpha_{s_1}^{L_\nu}&=\max\left\{W_{ac}^{L_\nu}(Y)\,|\,Y\textit{ is a SAC in }G\textit{ with }|Y|\geq\left\lceil\frac{n}{2}\right\rceil\right\},\\ \alpha_{s_1}^{U_\nu}&=\max\left\{W_{ac}^{U_\nu}(Y)\,|\,Y\textit{ is a SAC in }G\textit{ with }|Y|\geq\left\lceil\frac{n}{2}\right\rceil\right\}.\end{aligned} \tag{27}$$

Proof. Since G is a CIVIFG, so all arcs are SA, and each vertex is neighbor to all others vertices. Also, the number of arcs in SAC of both G and G^* is identical because each arc in both graphs is strong. Now, the SACN of G^* is $\lceil n/2\rceil$. Therefore, the minimum number of arcs in an SAC of G is $\lceil n/2\rceil$. This completes the proof. □

Theorem 15. *If K_{σ_1,σ_2} is a CB-IVIFG with partite set V_1 and V_2. Then,*

$$\begin{aligned}\alpha_{s_1}^{L_\mu}(K_{\sigma_1,\sigma_2})&=\min\left\{W_{ac}^{L_\mu}(Y)\,\middle|\,Y\textit{ is a SAC in }K_{\sigma_1,\sigma_2}\textit{ with }|Y|\right.\\ &\qquad\left.\geq\max\{|V_1|,|V_2|\}\right\},\\ \alpha_{s_1}^{U_\mu}(K_{\sigma_1,\sigma_2})&=\min\left\{W_{ac}^{U_\mu}(Y)\,\middle|\,Y\textit{ is a SAC in }K_{\sigma_1,\sigma_2}\textit{ with }|Y|\right.\\ &\qquad\left.\geq\max\{|V_1|,|V_2|\}\right\},\\ \alpha_{s_1}^{L_\nu}(K_{\sigma_1,\sigma_2})&=\max\{W_{ac}^{L_\nu}(Y)\,|\,Y\textit{ is a SAC in }K_{\sigma_1,\sigma_2}\textit{ with }|Y|\\ &\qquad\geq\max\{|V_1|,|V_2|\}\},\\ \alpha_{s_1}^{U_\nu}(K_{\sigma_1,\sigma_2})&=\max\{W_{ac}^{U_\nu}(Y)\,|\,Y\textit{ is a SAC in }K_{\sigma_1,\sigma_2}\textit{ with }|Y|\\ &\qquad\geq\max\{|V_1|,|V_2|\}\}.\end{aligned} \tag{28}$$

Proof. In K_{σ_1,σ_2}, all arcs are strong. Also, each node in V_1 is neighbor with all nodes in V_2 and contrariwise. Also, the number of arcs in an SAC of both G and G^* is identical because each arc in both graph is strong. Now, the arc

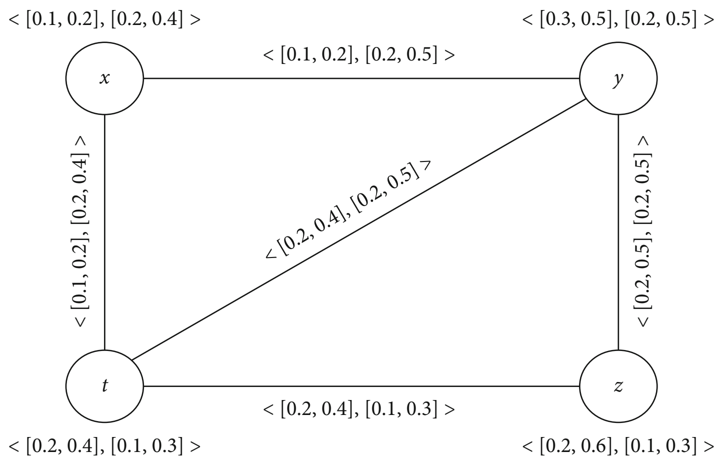

Figure 1: An IVIFG G for a strong node cover (SNC).

Table 2: Calculating the weight of SISs.

D	$W_{nc}^{L_\mu}(D)$	$W_{nc}^{U_\mu}(D)$	$W_{nc}^{L_\nu}(D)$	$W_{nc}^{U_\nu}(D)$	$W_{nc}(D)$
$\{y, t\}$	$0.1 + 0.1$	$0.2 + 0.2$	$0.2 + 0.2$	$0.5 + 0.5$	$\langle[0.2,0.4], [0.4,1]\rangle$
$\{x, y, z\}$	$0.1 + 0.1 + 0.2$	$0.2 + 0.2 + 0.4$	$0.2 + 0.2 + 0.2$	$0.5 + 0.5 + 0.5$	$\langle[0.4,0.8], [0.6,1.5]\rangle$
$\{x, z, t\}$	$0.1 + 0.1 + 0.2$	$0.2 + 0.2 + 0.4$	$0.2 + 0.2 + 0.2$	$0.5 + 0.5 + 0.5$	$\langle[0.4,0.8], [0.6,1.5]\rangle$
$\{y, z, t\}$	$0.1 + 0.2 + 0.1$	$0.2 + 0.4 + 0.2$	$0.2 + 0.2 + 0.2$	$0.5 + 0.5 + 0.5$	$\langle[0.4,0.8], [0.6,1.5]\rangle$
$\{x, y, t\}$	$0.1 + 0.1 + 0.1$	$0.2 + 0.2 + 0.2$	$0.2 + 0.2 + 0.2$	$0.5 + 0.5 + 0.5$	$\langle[0.3,0.6], [0.6,1.5]\rangle$
$\{x, y, z, t\}$	$0.1 + 0.1 + 0.2 + 0.1$	$0.2 + 0.2 + 0.4 + 0.2$	$0.2 + 0.2 + 0.2 + 0.2$	$0.5 + 0.5 + 0.5 + 0.5$	$\langle[0.5,1], [0.8,2]\rangle$

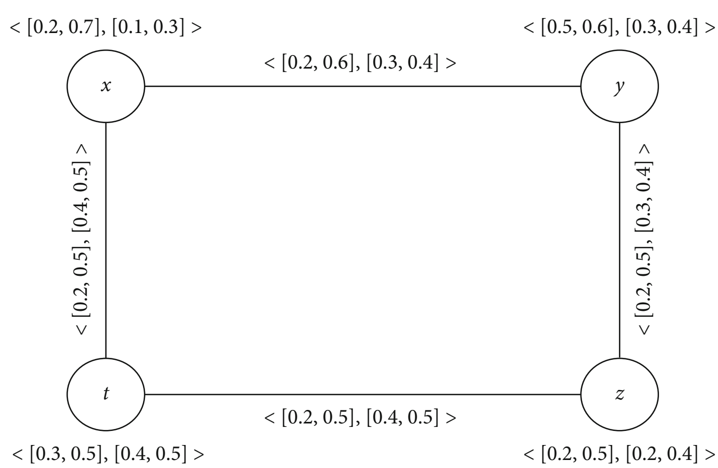

Figure 2: A strong IVIFG G for a strong independent set (SIS).

Table 3: Calculating the weight of SISs.

D	$W_{nc}^{L_\mu}(D)$	$W_{nc}^{U_\mu}(D)$	$W_{nc}^{L_\nu}(D)$	$W_{nc}^{U_\nu}(D)$	$W_{nc}(D)$
$\{x, t\}$	$0.2 + 0.2$	$0.5 + 0.4$	$0.4 + 0.4$	$0.5 + 0.5$	$\langle[0.4,0.9], [0.8,1]\rangle$
$\{y, t\}$	$0.2 + 0.2$	$0.5 + 0.5$	$0.3 + 0.4$	$0.4 + 0.5$	$\langle[0.4,1], [0.7,0.9]\rangle$

covering number of $K^*_{\sigma_1,\sigma_2}$ is max $\{|V_1|, |V_2|\}$. Therefore, the minimum number of arcs in an SAC of K_{σ_1,σ_2} is max $\{|V_1|, |V_2|\}$. Thus, the result is obtained. □

Definition 16. Let G be an IVIFG. A set T of SAs in G so that no two arcs in T have a common node is named an SIS of arcs or an SM in G.

Definition 17. Let T be an SM in IVIFG G. If $xy \in T$, then, we say that T strongly matches x to y. The weight of an SM is described as

$$W_{sm}(T) = \left\langle \left[W^{L_\mu}_{sm}(T), W^{U_\mu}_{sm}(T)\right], \left[W^{L_\nu}_{sm}(T), W^{U_\nu}_{sm}(T)\right] \right\rangle,$$
$$W_{sm}(T) = \left\langle \left[\sum_{xy\in T} \mu^L_B(xy), \sum_{xy\in T} \mu^U_B(xy)\right], \left[\sum_{xy\in T} \nu^L_B(xy), \sum_{xy\in T} \nu^U_B(xy)\right] \right\rangle. \tag{29}$$

An SMN of an IVIFG G is shown by $\beta_{s_1}(G) = \beta_{s_1} = \langle [\beta^{L_\mu}_{s_1}, \beta^{U_\mu}_{s_1}], [\beta^{L_\nu}_{s_1}, \beta^{U_\nu}_{s_1}] \rangle$, which

$$\begin{aligned}
\beta^{L_\mu}_{s_1} &= \max\left\{ W^{L_\mu}_{sm}(T) \middle| T \text{ is the SM of } G \right\},\\
\beta^{U_\mu}_{s_1} &= \max\left\{ W^{U_\mu}_{sm}(T) \middle| T \text{ is the SM of } G \right\},\\
\beta^{L_\nu}_{s_1} &= \min\left\{ W^{L_\nu}_{sm}(T) \middle| T \text{ is the SM of } G \right\},\\
\beta^{U_\nu}_{s_1} &= \min\left\{ W^{U_\nu}_{sm}(T) \middle| T \text{ is the SM of } G \right\}.
\end{aligned} \tag{30}$$

A maximum SM in an IVIFG G is an SM of maximum IVMBs and minimum IVNMBs.

Theorem 18. *If G is a CIVIFG, then*

$$\begin{aligned}
\beta^{L_\mu}_{s_1} &= \max\left\{ W^{L_\mu}_{sm}(T) \middle| T \textit{ is a SM with } |T| \le \left\lfloor \frac{n}{2} \right\rfloor \right\},\\
\beta^{U_\mu}_{s_1} &= \max\left\{ W^{U_\mu}_{sm}(T) \middle| T \textit{ is a SM with } |T| \le \left\lfloor \frac{n}{2} \right\rfloor \right\},\\
\beta^{L_\nu}_{s_1} &= \min\left\{ W^{L_\nu}_{sm}(T) \middle| T \textit{ is a SM with } |T| \le \left\lfloor \frac{n}{2} \right\rfloor \right\},\\
\beta^{U_\nu}_{s_1} &= \min\left\{ W^{U_\nu}_{sm}(T) \middle| T \textit{ is a SM with } |T| \le \left\lfloor \frac{n}{2} \right\rfloor \right\}.
\end{aligned} \tag{31}$$

Proof. Since G is a CIVIFG, all arcs are strong, and each node is neighbor to all other nodes. Also, the number of arcs in an SM of both G and G^* is identical because each arc in both graph is strong. Now, the SMN of G^* is $\lfloor n/2 \rfloor$. Therefore, the maximum number of arcs in an SM of G is $\lfloor n/2 \rfloor$. Hence, the result follows. □

Theorem 19. *For a CB-IVIFG K_{σ_1,σ_2} with partite set V_1 and V_2,*

$$\begin{aligned}
\beta^{L_\mu}_{s_1}(K_{\sigma_1,\sigma_2}) &= \max\left\{ W^{L_\mu}_{sm}(T) \middle| T \textit{ is a SM in } K_{\sigma_1,\sigma_2} \textit{ with } |T| \le \min\{|V_1|, |V_2|\} \right\},\\
\beta^{U_\mu}_{s_1}(K_{\sigma_1,\sigma_2}) &= \max\left\{ W^{U_\mu}_{sm}(T) \middle| T \textit{ is a SM in } K_{\sigma_1,\sigma_2} \textit{ with } |T| \le \min\{|V_1|, |V_2|\} \right\},\\
\beta^{L_\nu}_{s_1}(K_{\sigma_1,\sigma_2}) &= \min\left\{ W^{L_\nu}_{sm}(T) \middle| T \textit{ is a SM in } K_{\sigma_1,\sigma_2} \textit{ with } |T| \le \min\{|V_1|, |V_2|\} \right\},\\
\beta^{U_\nu}_{s_1}(K_{\sigma_1,\sigma_2}) &= \min\left\{ W^{U_\nu}_{sm}(T) \middle| T \textit{ is a SM in } K_{\sigma_1,\sigma_2} \textit{ with } |T| \le \min\{|V_1|, |V_2|\} \right\}.
\end{aligned} \tag{32}$$

Proof. In K_{σ_1,σ_2}, all arcs are strong. Also, each node in V_1 is neighbor with all nodes in V_2 and contrariwise. Thus, the number of arcs in an SM of both K_{σ_1,σ_2} and $K^*_{\sigma_1,\sigma_2}$ is identical because each arc in both graphs is strong. Now, the SMN of $K^*_{\sigma_1,\sigma_2}$ is max $\{|V_1|, |V_2|\}$. Therefore, the maximum number of arcs in an SM of K_{σ_1,σ_2} is max $\{|V_1|, |V_2|\}$.

Hence, the result is obtained. □

Example 3. Consider a strong IVIFG G is drawn in Figure 3. All arcs are strong, and the SACs are as follows:

$$\begin{aligned}
Y_1 &= \{xy, tz\},\\
Y_2 &= \{xt, yz\},\\
Y_3 &= \{yt, tx, yz\},\\
Y_4 &= \{yt, xy, tz\},\\
Y_5 &= \{xy, yz, zt\},\\
Y_6 &= \{xy, xt, tz\},\\
Y_7 &= \{xt, xy, yz\},\\
Y_8 &= \{xt, zt, yz\}.
\end{aligned} \tag{33}$$

The calculation of the weight of SACs is shown in Table 4.

So, $\alpha_{s_1} = \langle [0.3,0.6], [0.6,1.4] \rangle$.

Again, the two sets Y_1 and Y_2 are the only SAC and SM in G. So,

$$\begin{aligned}
W_{sm}(Y_1) &= \langle [0.3,0.6], [0.3,0.8] \rangle,\\
W_{sm}(Y_2) &= \langle [0.3,0.7], [0.4,0.9] \rangle.
\end{aligned} \tag{34}$$

Hence, $\beta_{s_1} = \langle [0.3,0.7], [0.3,0.8] \rangle$.

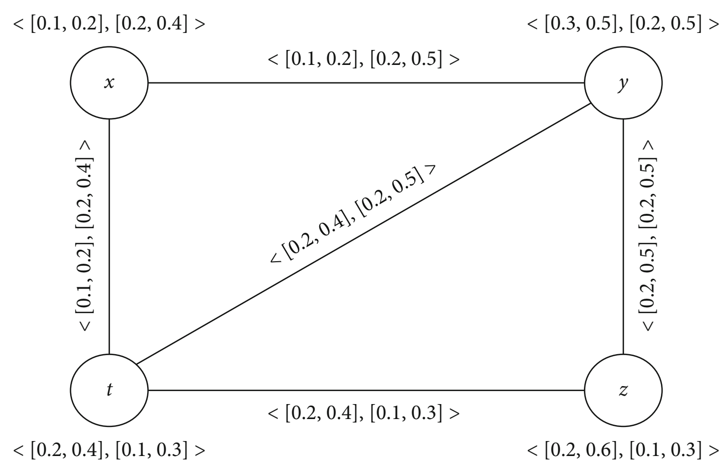

FIGURE 3: A strong IVIFG G for strong maching (SM).

TABLE 4: Calculating the weight of strong arc cover sets Y.

Y	$W^{L_\mu}_{ac}(Y)$	$W^{U_\mu}_{ac}(Y)$	$W^{L_\nu}_{ac}(Y)$	$W^{U_\nu}_{ac}(Y)$	$W_{ac}(Y)$
Y_1	$0.1+0.2$	$0.2+0.4$	$0.2+0.1$	$0.5+0.3$	$\langle[0.3,0.6],[0.3,0.8]\rangle$
Y_2	$0.1+0.2$	$0.2+0.5$	$0.2+0.2$	$0.4+0.5$	$\langle[0.3,0.7],[0.4,0.9]\rangle$
Y_3	$0.2+0.1+0.2$	$0.4+0.2+0.5$	$0.2+0.2+0.2$	$0.5+0.4+0.5$	$\langle[0.5,1.1],[0.6,1.4]\rangle$
Y_4	$0.2+0.1+0.2$	$0.4+0.2+0.4$	$0.2+0.2+0.1$	$0.5+0.5+0.3$	$\langle[0.5,1],[0.5,1.3]\rangle$
Y_5	$0.1+0.2+0.2$	$0.2+0.5+0.4$	$0.2+0.2+0.1$	$0.5+0.5+0.3$	$\langle[0.5,1.1],[0.5,1.3]\rangle$
Y_6	$0.1+0.1+0.2$	$0.2+0.2+0.4$	$0.2+0.2+0.1$	$0.5+0.4+0.3$	$\langle[0.4,0.8],[0.5,1.2]\rangle$
Y_7	$0.1+0.1+0.2$	$0.2+0.2+0.5$	$0.2+0.2+0.2$	$0.4+0.5+0.5$	$\langle[0.4,0.9],[0.6,1.4]\rangle$
Y_8	$0.1+0.2+0.2$	$0.2+0.4+0.5$	$0.2+0.1+0.2$	$0.4+0.3+0.5$	$\langle[0.4,1.1],[0.5,1.2]\rangle$

Example 4. Consider an IVIFG G is drawn in Figure 4.

All SAs are xt, zt, and yz, and all SACs are as follows:

$$\begin{aligned} Y_1 &= \{xt, yz\}, \\ Y_2 &= \{xt, tz, yz\}. \end{aligned} \tag{35}$$

The calculation of the weight of SACs is shown in Table 5. So, $\alpha_{s_1} = \langle[0.4,1],[1.1,1.4]\rangle$.

The set $Y_1 = \{xt, yz\}$ is the only SIAC. So, $\beta_{s_1} = W_{sm}(Y_1) = \langle[0.4,1],[0.7,0.9]\rangle$.

Theorem 20. *Let G be an IVIFG containing no IN. Then,*

$$\begin{aligned} \alpha^{L_\mu}_{s_0} + \beta^{L_\mu}_{s_0} &= W^{L_\mu}(V), \\ \alpha^{L_\nu}_{s_0} + \beta^{L_\nu}_{s_0} &= W^{L_\nu}(V), \\ \alpha^{U_\mu}_{s_0} + \beta^{U_\mu}_{s_0} &= W^{U_\mu}(V), \\ \alpha^{U_\nu}_{s_0} + \beta^{U_\nu}_{s_0} &= W^{U_\nu}(V). \end{aligned} \tag{36}$$

Proof. Let M_{s_0} be a minimum SNC of G, which

$$\begin{aligned} \alpha^{L_\mu}_{s_0} &= W^{L_\mu}\left(M_{s_0}\right), \\ \alpha^{L_\nu}_{s_0} &= W^{L_\nu}\left(M_{s_0}\right), \\ \alpha^{U_\mu}_{s_0} &= W^{U_\mu}\left(M_{s_0}\right), \\ \alpha^{U_\nu}_{s_0} &= W^{U_\nu}\left(M_{s_0}\right). \end{aligned} \tag{37}$$

Then, $V - M_{s_0}$ is an SIS of nodes. In other words, the nodes in $V - M_{s_0}$ are incident on SAs of G. Thus,

$$\begin{aligned} \beta^{L_\mu}_{s_0} \geq W^{L_\mu}\left(V - M_{s_0}\right) = W^{L_\mu}(V) - \alpha^{L_\mu}_{s_0} &\Rightarrow \alpha^{L_\mu}_{s_0} + \beta^{L_\mu}_{s_0} \geq W^{L_\mu}(V), \\ \beta^{U_\mu}_{s_0} \geq W^{U_\mu}\left(V - M_{s_0}\right) = W^{U_\mu}(V) - \alpha^{U_\mu}_{s_0} &\Rightarrow \alpha^{U_\mu}_{s_0} + \beta^{U_\mu}_{s_0} \geq W^{U_\mu}(V), \\ \beta^{L_\nu}_{s_0} \leq W^{L_\nu}\left(V - M_{s_0}\right) = W^{L_\nu}(V) - \alpha^{L_\nu}_{s_0} &\Rightarrow \alpha^{L_\nu}_{s_0} + \beta^{L_\nu}_{s_0} \leq W^{L_\nu}(V), \\ \beta^{U_\nu}_{s_0} \leq W^{U_\nu}\left(V - M_{s_0}\right) = W^{U_\nu}(V) - \alpha^{U_\nu}_{s_0} &\Rightarrow \alpha^{U_\nu}_{s_0} + \beta^{U_\nu}_{s_0} \leq W^{U_\nu}(V). \end{aligned} \tag{38}$$

Let $\beta^{L_\mu}_{s_0} = W(Q_{s_0})$, where Q_{s_0} is a maximum SIS of nodes in G. That is, no two nodes in Q_{s_0} are neighbor to each other

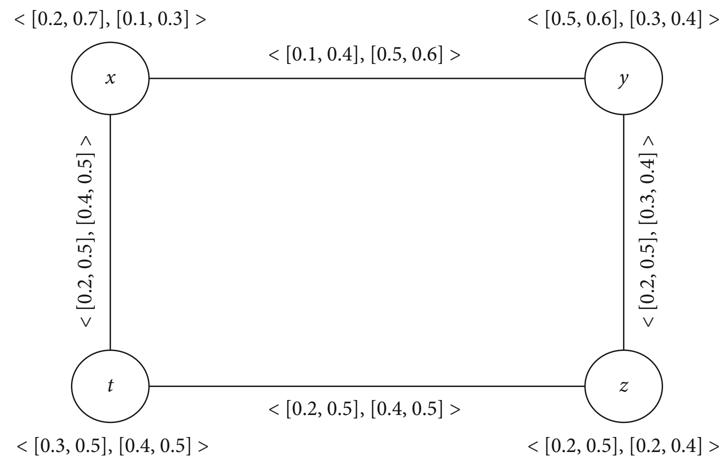

Figure 4: An IVIFG G for strong independent arc cover (SIAC).

Table 5: Calculating the weight of strong arc cover sets Y.

Y	$W_{ac}^{L_\mu}(Y)$	$W_{ac}^{U_\mu}(Y)$	$W_{ac}^{L_\nu}(Y)$	$W_{ac}^{U_\nu}(Y)$	$W_{ac}(Y)$
Y_1	$0.2+0.2$	$0.5+0.5$	$0.4+0.3$	$0.5+0.4$	$\langle[0.4,1], [0.7,0.9]\rangle$
Y_2	$0.2+0.2+0.2$	$0.5+0.5+0.5$	$0.4+0.4+0.3$	$0.5+0.5+0.4$	$\langle[0.6,1.5], [1.1,1.4]\rangle$

by an SA, and thus, the node in $V-Q_{s_0}$ strongly covers all SAs of G. Hence, $V-Q_{s_0}$ is an SNC of G, and $\alpha_{s_0}^{L_\mu}$ and $\alpha_{s_0}^{U_\mu}$ are the minimum lower and upper of IVMBs, and $\alpha_{s_0}^{L_\nu}$ and $\alpha_{s_0}^{U_\nu}$ are the maximum lower and upper of IVNMBs. So,

$$\begin{aligned}
\alpha_{s_0}^{L_\mu} &\le W^{L_\mu}\left(V-Q_{s_0}\right) = W^{L_\mu}(V) - \beta_{s_0}^{L_\mu} \Rightarrow \alpha_{s_0}^{L_\mu} + \beta_{s_0}^{L_\mu} \le W^{L_\mu}(V),\\
\alpha_{s_0}^{U_\mu} &\le W^{U_\mu}\left(V-Q_{s_0}\right) = W^{U_\mu}(V) - \beta_{s_0}^{U_\mu} \Rightarrow \alpha_{s_0}^{U_\mu} + \beta_{s_0}^{U_\mu} \le W^{U_\mu}(V),\\
\alpha_{s_0}^{L_\nu} &\ge W^{L_\nu}\left(V-Q_{s_0}\right) = W^{L_\nu}(V) - \beta_{s_0}^{L_\nu} \Rightarrow \alpha_{s_0}^{L_\nu} + \beta_{s_0}^{L_\nu} \ge W^{L_\nu}(V),\\
\alpha_{s_0}^{U_\nu} &\ge W^{U_\nu}\left(V-Q_{s_0}\right) = W^{U_\nu}(V) - \beta_{s_0}^{U_\nu} \Rightarrow \alpha_{s_0}^{U_\nu} + \beta_{s_0}^{U_\nu} \ge W^{U_\nu}(V).
\end{aligned} \tag{39}$$

From (38) and (39), we have

$$\begin{aligned}
&\alpha_{s_0}^{L_\mu} + \beta_{s_0}^{L_\mu} = W^{L_\mu}(V), \alpha_{s_0}^{U_\mu} + \beta_{s_0}^{U_\mu} = W^{U_\mu}(V),\\
&\alpha_{s_0}^{L_\nu} + \beta_{s_0}^{L_\nu} = W^{L_\nu}(V), \alpha_{s_0}^{U_\nu} + \beta_{s_0}^{U_\nu} = W^{U_\nu}(V).
\end{aligned} \tag{40}$$

□

Definition 21. Let G be an IVIFG and M be an SM in G. Then, M is named a PSM if M strongly matches each node of G to some nodes of G.

Example 5. Consider an IVIFG G is drawn in Figure 5. All arcs are strong, and the sets M_1 and M_2 are PSMs. The calculation of the weight of PSMs is given in Table 6.

$$\begin{aligned}
M_1 &= \{xt, yz\},\\
M_2 &= \{xz, yt\},\\
M_3 &= \{xt, xz, yz\},\\
M_4 &= \{xt, ty, yz\}.
\end{aligned} \tag{41}$$

So, $\beta_{s_1} = \langle[0.6,1.3], [0.7,0.9]\rangle$.

Hence, $\alpha_{s_1} = \langle[0.3,0.7], [1.2,1.6]\rangle$.

Now, we introduced PD in IVIFGs using SAs based on PSM. Also, some useful results are established.

Definition 22. A set D of nodes of IVIFG G is an SDS of G if every node of $V-D$ is a strong neighbor of some nodes in D.

Definition 23. The weight of an SDS D is defined as

$$W_{sd}(D) = \left\langle \left[W_{sd}^{L_\mu}(D), W_{sd}^{U_\mu}(D)\right], \left[W_{sd}^{L_\nu}(D), W_{sd}^{U_\nu}(D)\right] \right\rangle, \tag{42}$$

or

$$W_{sd}(D) = \left\langle \left[\sum_{x\in D} \mu_B^L(xy), \sum_{x\in D} \mu_B^U(xy)\right], \left[\sum_{x\in D} \nu_B^L(xy), \sum_{x\in D} \nu_B^U(xy)\right] \right\rangle, \tag{43}$$

where $\mu_B^L(xy)$ and $\mu_B^U(xy)$ are the minimum lower and upper of IVMBs and $\nu_B^L(xy)$ and $\nu_B^U(xy)$ are the maximum lower and upper of IVNMBs of SAs incident on x, respectively.

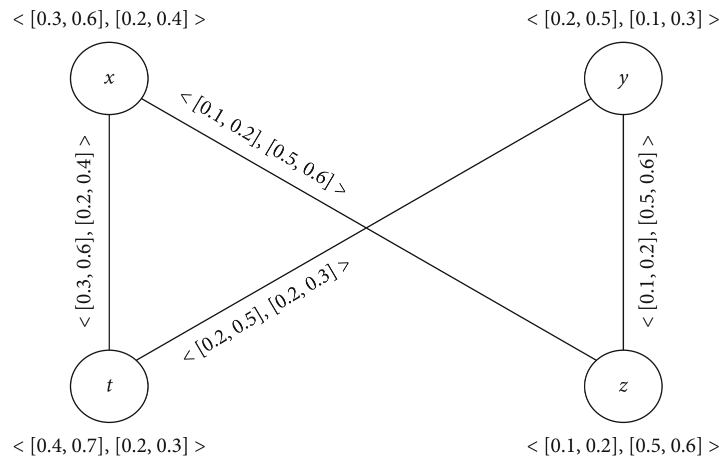

Figure 5: An IVIFG G for perfect strong matching (PSM).

Table 6: Calculating the weight of perfect strong matchings M.

M	$W_{sm}^{L_\mu}(M)$	$W_{sm}^{U_\mu}(M)$	$W_{sm}^{L_\nu}(M)$	$W_{sm}^{U_\nu}(M)$	$W_{sm}(M)$
M_1	$0.3+0.1$	$0.6+0.2$	$0.2+0.5$	$0.4+0.6$	$\langle[0.4,0.8],[0.7,1]\rangle$
M_2	$0.1+0.2$	$0.2+0.5$	$0.5+0.2$	$0.6+0.3$	$\langle[0.3,0.7],[0.7,0.9]\rangle$
M_3	$0.3+0.1+0.1$	$0.6+0.2+0.2$	$0.2+0.5+0.5$	$0.4+0.6+0.6$	$\langle[0.5,1],[1.2,1.6]\rangle$
M_4	$0.3+0.2+0.1$	$0.6+0.5+0.2$	$0.2+0.2+0.5$	$0.4+0.3+0.6$	$\langle[0.6,1.3],[0.9,1.3]\rangle$

An SDN of an IVIFG G is denoted by $\gamma_s(G)=\gamma_s=\langle[\gamma_s^{L_\mu},\gamma_s^{U_\mu}],[\gamma_s^{L_\nu},\gamma_s^{U_\nu}]\rangle$, where

$$\begin{aligned}
\gamma_s^{L_\mu} &= \min\left\{W_{sd}^{L_\mu}(D)\,\middle|\,D \text{ is the SDSs of } G\right\},\\
\gamma_s^{U_\mu} &= \min\left\{W_{sd}^{U_\mu}(D)\,\middle|\,D \text{ is the SDSs of } G\right\},\\
\gamma_s^{L_\nu} &= \max\left\{W_{sd}^{L_\nu}(D)\,\middle|\,D \text{ is the SDSs of } G\right\},\\
\gamma_s^{U_\nu} &= \max\left\{W_{sd}^{U_\nu}(D)\,\middle|\,D \text{ is the SDSs of } G\right\}.
\end{aligned} \tag{44}$$

Definition 24. Let G be an IVIFG. A set D of nodes is named to be an SPDS if D is an SDS and the IVIF-subgraph induced by D has a PSM. The weight of an SPDS D is described as $W_{spd}(D)=\langle[W_{spd}^{L_\mu}(D),W_{spd}^{U_\mu}(D)],[W_{spd}^{L_\nu}(D),W_{spd}^{U_\nu}(D)]\rangle$, which

$$W_{spd}(D)=\left\langle\left[\sum_{x\in D}\mu_B^L(xy),\sum_{x\in D}\mu_B^U(xy)\right],\left[\sum_{x\in D}\nu_B^L(xy),\sum_{x\in D}\nu_B^U(xy)\right]\right\rangle. \tag{45}$$

An SPDN of an IVIFG G is denoted by $\gamma_{spd}(G)=\gamma_{spd}=\langle[\gamma_{spd}^{L_\mu},\gamma_{spd}^{U_\mu}],[\gamma_{spd}^{L_\nu},\gamma_{spd}^{U_\nu}]\rangle$, that

$$\begin{aligned}
\gamma_{spd}^{L_\mu} &= \min\left\{W_{spd}^{L_\mu}(D)\,\middle|\,D \text{ is the SPDSs of } G\right\},\\
\gamma_{spd}^{U_\mu} &= \min\left\{W_{spd}^{U_\mu}(D)\,\middle|\,D \text{ is the SPDSs of } G\right\},\\
\gamma_{spd}^{L_\nu} &= \max\left\{W_{spd}^{L_\nu}(D)\,\middle|\,D \text{ is the SPDSs of } G\right\},\\
\gamma_{spd}^{U_\nu} &= \max\left\{W_{spd}^{U_\nu}(D)\,\middle|\,D \text{ is the SPDSs of } G\right\}.
\end{aligned} \tag{46}$$

Example 6. Consider an IVIFG G is drawn in Figure 6. All SAs are xt, zt, and yz. The PDs in G are D_1, D_2, and D_3. The weights of these sets are calculated as follows:

$$\begin{aligned}
D_2 &= \{x,y\},\\
D_2 &= \{z,t\},\\
D_3 &= \{x,y,z,t\}.
\end{aligned} \tag{47}$$

Table 7 shows the calculation of the weight of PDs. Hence, $\gamma_{spd}(G)=\langle[0.4,1],[1.4,1.9]\rangle$.

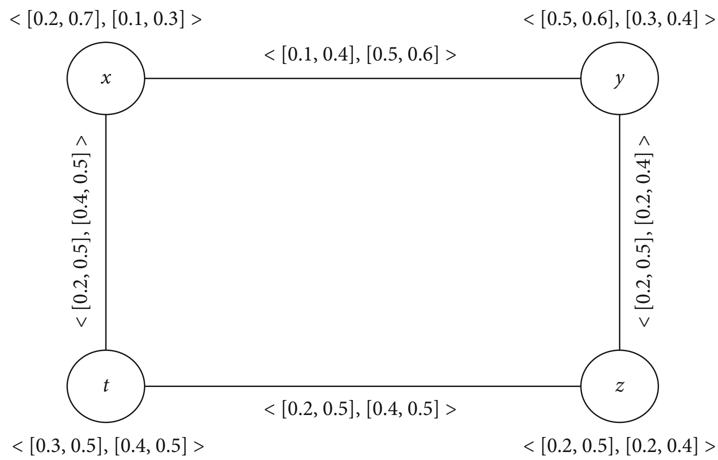

Figure 6: An IVIFG G for strong paired dominating set (SPDS).

Table 7: Calculating the weight of paired dominating sets.

D	$W^{L_\mu}_{spd}(D)$	$W^{U_\mu}_{spd}(D)$	$W^{L_\nu}_{spd}(D)$	$W^{U_\nu}_{spd}(D)$	$W_{spd}(D)$
D_1	$0.2 + 0.2$	$0.5 + 0.5$	$0.4 + 0.2$	$0.5 + 0.4$	$\langle[0.4,1], [0.6,0.9]\rangle$
D_2	$0.2 + 0.2$	$0.5 + 0.5$	$0.4 + 0.4$	$0.5 + 0.5$	$\langle[0.4,1], [0.8,1]\rangle$
D_3	$0.2 + 0.2 + 0.2 + 0.2$	$0.5 + 0.5 + 0.5 + 0.5$	$0.4 + 0.2 + 0.4 + 0.4$	$0.5 + 0.4 + 0.5 + 0.5$	$\langle[0.8,2], [1.4,1.9]\rangle$

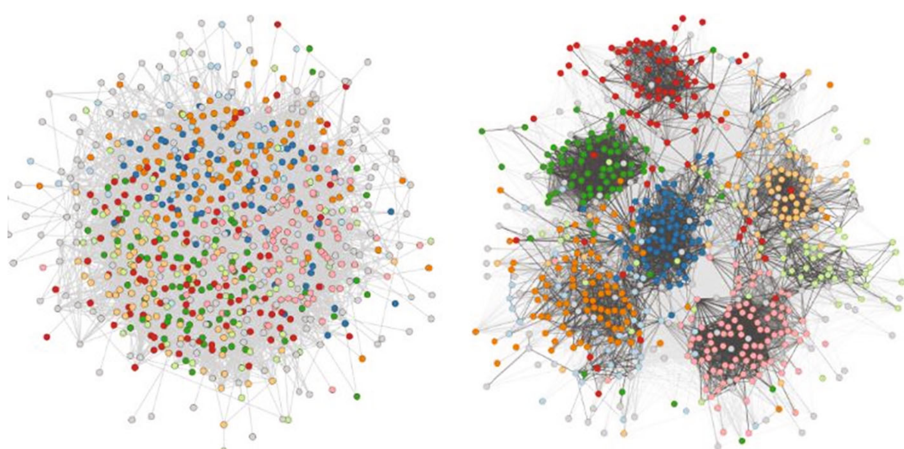

Figure 7: Scientific community network of researchers.

Theorem 25. *Let G be a CIVIFG. Then,*

$$\begin{aligned}\gamma^{L_\mu}_{spd} &= 2\mu^L_B(xy),\\ \gamma^{L_\nu}_{spd} &= 2\nu^L_B(xy),\\ \gamma^{U_\mu}_{spd} &= 2\mu^U_B(xy),\\ \gamma^{U_\nu}_{spd} &= 2\nu^U_B(xy),\end{aligned} \tag{48}$$

where $\mu^L_B(xy)$ and $\mu^U_B(xy)$ are the lower and upper of IVMB and $\nu^L_B(xy)$ and $\nu^U_B(xy)$ are the lower and upper of IVNMB of any weakest arc in G, respectively.

Proof. Since G is a CIVIFG, all arcs are strong, and every vertex is neighbor to all other vertices. Then, any set consisting of two nodes $\{x_1, x_2\}$ in G forms an SPDS. Hence,

$$\begin{aligned}\gamma^{L_\mu}_{spd} &= \mu^L_B(xy) + \mu^L_B(xy) = 2\mu^L_B(xy),\\ \gamma^{U_\mu}_{spd} &= \mu^U_B(xy) + \mu^U_B(xy) = 2\mu^U_B(xy),\\ \gamma^{L_\nu}_{spd} &= \nu^L_B(xy) + \nu^L_B(xy) = 2\nu^L_B(xy),\\ \gamma^{U_\nu}_{spd} &= \nu^U_B(xy) + \nu^U_B(xy) = 2\nu^U_B(xy),\end{aligned} \tag{49}$$

where xy is the weakest arc in G. □

Table 8: Data set.

Vertices	Communities	Number of members	Average attendance of members
a	Biology	35	30
b	Chemistry	25	21
c	Engineering	55	45
d	Information technology (IT)	75	60
e	Mathematics	40	32
f	Medicine	50	45
g	Physics	30	24
h	Social sciences	45	37

Table 9: The IVIFNs of scientific communities.

Vertices	Communities	The IVIFNs corresponding to each community.
a	Biology	$\langle[0.80,0.90],[0.05,0.10]\rangle$
b	Chemistry	$\langle[0.79,0.89],[0.11,0.21]\rangle$
c	Engineering	$\langle[0.76,0.86],[0.14,0.16]\rangle$
d	Information technology (IT)	$\langle[0.75,0.85],[0.10,0.15]\rangle$
e	Mathematics	$\langle[0.75,0.85],[0.10,0.15]\rangle$
f	Medicine	$\langle[0.80,0.90],[0.5,0.10]\rangle$
g	Physics	$\langle[0.75,0.85],[0.10,0.15]\rangle$
h	Social sciences	$\langle[0.77,0.87],[0.10,0.13]\rangle$

4. Application

Social networks are a group of individuals or organizations with common tastes or interests that come together to achieve specific goals. Each member is named an actor. Social networks are characterized by complex relationships and interactions between actors. The main reasons for creating social networks are individual relationships, labor relations, scientific relations, shared tastes, interests and hobbies, sociopolitical motives, and virtual network analysis.

Graphs are used as a mathematical tool to represent and analyze a social network by visually representing social networks. In these graphs, the actors are considered as vertices of the graph, and the connections between them are displayed by the edges of the graph. Intuitively, the edges are distributed on social networks locally. This means that the number of edges distributed among a group of vertices is much greater than the number of distribution edges among this group of vertices and the rest of the vertices of the graph. This feature, which can be seen in graphs related to real data, is called a community. In some sources, the community is also called a cluster or module. In other words, communities are a set of vertices that are more likely to share common features than the rest of the graph. Since people in forums on a social network are more likely to have common interests, this information can be used to promote specific products by finding their interests. Most online social networks have overlapping communities. This means that these networks are made up of overlapping communities, and one vertex can belong to more than one community. Figure 7 illustrates the social network of researchers in a country that is a member of different scientific communities according to the subjects under study. These communities include chemistry, biology, engineering, information technology (IT), mathematics, medicine, physics, and social sciences. Table 8 shows the number of members of each community and the average number of members present at the meetings.

In the evaluations made by the members on the effect of community on the scientific promotion of members, since the mentioned variables have uncertain values, so for each community, we considered an interval-valued intuitionistic fuzzy number as the amount of influence of community on its members. Since the presence of

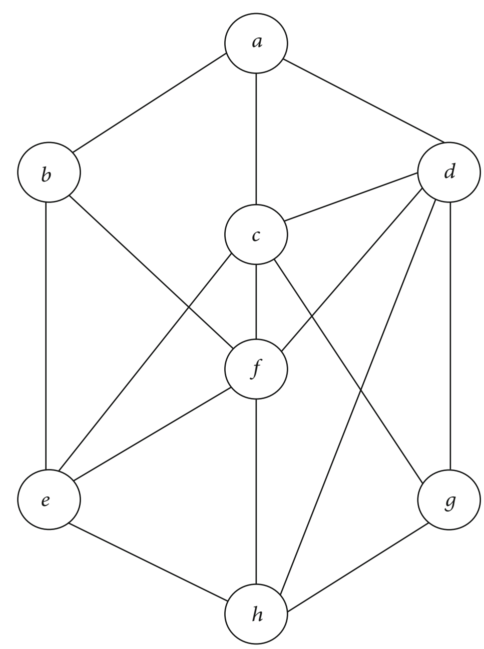

Figure 8: The scientific communities IVIFG.

Table 10: IVIFNs of relations between scientific communities.

Edges	IVIFNs	Edges	IVIFNs
ab	$\langle[0.97,0.89],[0.11,0.21]\rangle$	df	$\langle[0.75,0.85],[0.10,0.15]\rangle$
ac	$\langle[0.76,0.86],[0.14,0.16]\rangle$	dg	$\langle[0.75,0.85],[0.10,0.15]\rangle$
ad	$\langle[0.75,0.85],[0.10,0.15]\rangle$	dh	$\langle[0.75,0.85],[0.10,0.15]\rangle$
be	$\langle[0.75,0.85],[0.11,0.21]\rangle$	ef	$\langle[0.75,0.85],[0.10,0.15]\rangle$
bf	$\langle[0.79,0.89],[0.11,0.21]\rangle$	eh	$\langle[0.75,0.85],[0.10,0.15]\rangle$
cd	$\langle[0.75,0.85],[0.14,0.16]\rangle$	fh	$\langle[0.77,0.87],[0.10,0.13]\rangle$
cf	$\langle[0.76,0.86],[0.14,0.16]\rangle$	gh	$\langle[0.75,0.85],[0.10,0.15]\rangle$
ce	$\langle[0.75,0.85],[0.14,0.16]\rangle$	cg	$\langle[0.75,0.85],[0.14,0.16]\rangle$

Step 1. Consider vertex x as a member of F. Then, remove all adjacent vertices of x.
Step 2. Consider another arbitrary vertex in the remaining graph as a new member of F.
Depending on which member of the remaining vertex set is selected, different independent sets, including x, are obtained.
Step 3. Repeat Step 2 to select all possible vertices.

Algorithm 1: Finding the maximal SISs F of G containing an arbitrary vertex x.

Table 11: Calculations for finding maximal SISs in the IVIFG of Figure 8.

Step 1	Step 2	Step 3	SISs
	e	g	{a,e,g}
	f	g	{a, f, g}
a	g	f	{a,g,f}
		e	{a,g,e}
	h		{a,h}
	c	h	{b,c,h}
b	d		{b,d}
	g		{b,g}
	h	c	{b,h,c}
c	b	h	{c,b,h}
	h	b	{c,h,b}
d	b		{d,b}
	e		{d,e}
	a	g	{e,a,g}
e	d		{e,d}
	g	a	{e,g,a}
f	a	g	{f,a,g}
	g	a	{f,g,e}
	a	f	{g,a,f}
		e	{g,a,e}
g	b		{g,b}
	e	a	{g,e,a}
	f	a	{g,f,a}
	a		{h,a}
h	b	c	{h, b, c}
	c	b	{h,c,b}

members in the meetings of the community is effective on the scientific promotion of members, we introduced the ratio of the average number of members present in the meetings to the total number as an IVIFN. For example, studies have shown that the biology community is 80 to 90 percent effective in advancing the science of its members and 5 to 10 percent ineffective. These values are specified in Table 9.

The strong relationships between scientific communities are illustrated in the form of an IVIFG in Figure 8. In this IVIFG, the membership values of the edges are the effect that the members of the two communities have on their scientific advancement. For example, the collaboration between the two communities of chemistry and medicine is about 79 to 89 percent effective in the scientific advancement of the members of each community and 11 to 21 percent ineffective. These values are shown in Table 10.

In general, there is no polynomial algorithm for finding a maximum independent set for an arbitrary graph. This means that it is not possible to access such a collection in a short time. To obtain the maximal SISs in IVIFG with a small number of vertices, we use the following instructions.

Since all edges are SA, so by applying the above steps for all vertices on the IVIFG of Figure 8, all maximal SISs and cardinalities can be seen in Table 11. Now, by calculating the cardinal of all the SISs obtained from the above steps, we can also determine the maximum SISs.

The maximum SISs are $D_1 = \{a, e, g\}$, $D_2 = \{a, f, g\}$, and $D_3 = \{b, c, h\}$.

After calculating the weight of the above sets, we have $W(D_1) = \langle [2.25,2.55], [0.42,0.58] \rangle$, $W(D_2) = \langle [2.25,2.55], [0.42,0.53] \rangle$, and $W(D_3) = \langle [2.25,2.55], [0.35,0.52] \rangle$.

Therefore, D_3 has the maximum weight of membership and the minimum weight of nonmembership, so it can be chosen as the best option. It is interesting to know that D_3 also has the maximum number of members in scientific communities. That is, strong independent scientific communities include chemistry, engineering, and social sciences.

Suppose knowledge-based companies intend to organize an exhibition at the meeting place of scientific communities to acquaint researchers with their scientific products. Researchers at the knowledge-based companies can be members of various scientific communities. The goal is to hold as many exhibitions as possible at the same time provided that each knowledge-based company has a maximum of one exhibition in a specific time period and to hold another exhibition at different time intervals. In this case, the maximum independent set is the maximum number of exhibitions that can be held at one time in scientific communities.

5. Conclusion

Analysis of uncertain problems by IVIFG is important because it gives more integrity and flexibility to the system. An IVIFG, as an extension of FGs, has good capabilities in dealing with problems that cannot be explained by FGs. They have been able to have wide applications even in fields such as psychology and identifying people based on cancerous behaviors. In this paper, covering and matching have been defined in IVIFGs using strong arcs. These concepts are introduced as an interval-valued intuitionistic number. One of the advantages of this method is that the amount of defined parameters can be expressed and compared in terms of membership and nonmembership. Also, the concepts of SNC, SIN, SAC, and SM in IVIFGs are determined, and the relations among them have been obtained. Furthermore, we have introduced the PD and SPDN in CIVIFG and CB-IVIFG. Since the parameters being studied are interval values, comparisons of these parameters may be limited in an IVIFG. Finally, we have presented an application of IVIFG in social networks. In their future work, the authors try to study the concepts of m-polar IVIFGs.

Data Availability

No data were used in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Natural Science Foundation of Guangdong Province of China (2022A1515011468).

References

[1] A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning–I," *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[3] A. Roselfeld, "Fuzzy graphs," in *Fuzzy Sets and Their Applications*, L. A. Zadeh, K. S. Fu, and M. Shimura, Eds., pp. 77–95, Academic Press, New York, NY, USA, 1975.

[4] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[5] K. T. Atanassov, *Intuitionistic Fuzzy Sets*, Physica-Verlag, Heidelberg, New York, NY, USA, 1999.

[6] M. Akram and B. Davvaz, "Strong intuitionistic fuzzy graphs," *Univerzitet u Nišu*, vol. 26, pp. 177–196, 2012.

[7] R. Mahapatra, S. Samanta, T. Allahviranloo, and M. Pal, "Radio fuzzy graphs and assignment of frequency in radio stations," *Computational and Applied Mathematics*, vol. 38, no. 3, pp. 1–20, 2019.

[8] R. Mahapatra, S. Samanta, and M. Pal, "Applications of edge colouring of fuzzy graphs," *Informatica*, vol. 31, no. 2, pp. 313–330, 2020.

[9] R. Mahapatra, S. Samanta, M. Pal et al., "Colouring of COVID-19 affected region based on fuzzy directed graphs," *Computers, Materials & Continua*, vol. 68, no. 1, pp. 1219–1233, 2021.

[10] H. Rashmanlou and M. Pal, "Antipodal interval-valued fuzzy graphs," *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, vol. 3, pp. 107–130, 2013.

[11] H. Rashmanlou and M. Pal, "Balanced interval-valued fuzzy graph," *Journal of Physical Sciences*, vol. 17, pp. 43–57, 2013.

[12] S. Kosari, Y. Rao, H. Jiang, X. Liu, P. Wu, and Z. Shao, "Vague graph structure with application in medical diagnosis," *Symmetry*, vol. 12, no. 10, p. 1582, 2020.

[13] Z. Kou, S. Kosari, and M. Akhoundi, "A novel description on vague graph with application in transportation systems," *Journal of Mathematics*, vol. 2021, Article ID 4800499, 11 pages, 2021.

[14] K. K. Krishna, Y. Talebi, H. Rashmanlou, A. A. Talebi, and F. Mofidnakhaei, "New concept of cubic graph with application," *Journal of Multiple Valued Logic and Soft Computing*, vol. 33, pp. 135–154, 2019.

[15] A. A. Talebi, "Cayley fuzzy graphs on the fuzzy groups," *Computational and Applied Mathematics*, vol. 37, no. 4, pp. 4611–4632, 2018.

[16] A. A. Talebi and W. A. Dudek, "Operations on level graphs of bipolar fuzzy graphs," *Bulletin Academiel De Stiinte A Republic Moldova Mathematica*, vol. 2, no. 81, pp. 107–124, 2016.

[17] K. T. Atanassov, "New topological operator over intuitionistic fuzzy sets," *Journal of Computational and Cognitive Engineering*, 2022.

[18] B. Mathew, S. J. John, and H. Garg, "Vertex rough graphs," *Complex and Intelligent Systems*, vol. 6, no. 2, pp. 347–353, 2020.

[19] N. Jan, K. Ullah, T. Mahmood et al., "Some root level modifications in interval valued fuzzy graphs and their generalizations including neutrosophic graphs," *Mathematics*, vol. 7, no. 1, p. 72, 2019.

[20] M. G. Voskoglou, "A combined use of softs sets and grey numbers in decision making," *Journal of Computational and Cognitive Engineering*, 2022.

[21] R. Mahapatra, S. Samanta, M. Pal, and Q. Xin, "Link prediction in social networks by neutrosophic graph," *International Journal of Computational Intelligence Systems*, vol. 13, no. 1, pp. 1699–1713, 2020.

[22] R. Mahapatra, S. Samanta, and M. Pal, "Generalized neutrosophic planar graphs and its application," *Journal of Applied Mathematics and Computing*, vol. 65, no. 1, pp. 693–712, 2021.

[23] R. Mahapatra, S. Samanta, M. Pal, and Q. Xin, "RSM index: a new way of link prediction in social networks," *Journal of Intelligent and Fuzzy Systems*, vol. 37, no. 2, pp. 2137–2151, 2019.

[24] K. T. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[25] H. Rashmanlou and R. A. Borzooei, "New concepts of interval-valued intuitionistic (S, T)-fuzzy graphs," *Journal of Intelligent and Fuzzy Systems*, vol. 30, no. 4, pp. 1893–1901, 2016.

[26] M. Dgenci, "A new distance measure for interval valued intuitionistic fuzzy sets and its application to group decision making problems with incomplete weights information," *Applied Soft Computing*, vol. 41, pp. 120–134, 2016.

[27] Z. Zhang, M. Wang, Y. Hu, J. Yang, Y. Ye, and Y. Li, "A dynamic interval-valued intuitionistic fuzzy sets applied to pattern recognition," *Mathematical Problems in Engineering*, vol. 2013, Article ID 408012, 16 pages, 2013.

[28] J. Y. Ahn, K. S. Han, and C. D. Lee, "An application of interval-valued intuitionistic fuzzy sets for medicdl diagnosis of headache," *International Journal of Innovative Computing, Information and Control*, vol. 7, no. 5, pp. 2755–2762, 2011.

[29] J. Hongmei and W. Lianhua, "Interval-valued fuzzy subsemigroups and subgroups associated by interval-valued fuzzy graphs," in *2009 WRI Global Congress on Intelligent System*, pp. 484–487, Xiamen, China, 2009.

[30] M. Akram and W. A. Dudek, "Interval-valued fuzzy graphs," *Computers and Mathematics with Applications*, vol. 61, no. 2, pp. 289–299, 2011.

[31] M. Akram, N. O. Alshehri, and W. A. Dudek, "Certain types of interval-valued fuzzy graphs," *Journal of Applied Mathematics*, vol. 2013, Article ID 857070, 11 pages, 2013.

[32] S. N. Mishra and A. Pal, "Product of interval-valued intuitionistic fuzzy graph," *Annals of Pure and Applied Mathematics*, vol. 5, no. 1, pp. 37–46, 2013.

[33] A. M. Ismayil and A. M. Ali, "On strong interval-valued intuitionistic fuzzy graph," *International Journal of Fuzzy Mathematics and Systems*, vol. 4, no. 2, pp. 161–168, 2014.

[34] A. A. Talebi, H. Rashmanlou, and S. H. Sadati, "Interval-valued intuitionistic fuzzy competition graph," *Journal of Multiple-Valued Logic & Soft Computing*, vol. 34, pp. 335–364, 2020.

[35] A. A. Talebi, H. Rashmanlou, and S. H. Sadati, "New concepts on m-polar interval-valued intuitionistic fuzzy graph," *Journal of Applied and Engineering Mathematics*, vol. 10, no. 3, pp. 806–818, 2020.

[36] P. Xu, H. Guan, A. A. Talebi, M. Ghassemi, and H. Rashmanlou, "Certain concepts of interval-valued intuitionistic fuzzy graphs with an application," *Advances in Mathematical Physics*, vol. 2022, Article ID 6350959, 12 pages, 2022.

[37] S. Sahoo, M. Pal, H. Rashmanlou, and R. A. Borzooei, "Covering and paired domination in intuitionistic fuzzy graphs," *Journal of Intelligent and Fuzzy Systems*, vol. 33, no. 6, pp. 4007–4015, 2017.