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Stability of Multiple Knapsack Problems with Interval Capacities

Samir A. Abass[1] and Asmaa S. Abdallah[1*]

[1]*Atomic Energy Authority, Nuclear Research Center, Cairo, Egypt.*

Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

In this study, the multiple knapsack problems (MKP) with uncertainty model is introduced. The uncertainty represents the capacities of the knapsack. A possibility degree of interval number is used to convert the uncertain capacities to deterministic capacities. Some basic stability notions in parametric multiple knapsack are defined. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind. A numerical example (case study) is introduced to present the suggested approach.

Keywords: Knapsack problem; interval number; stability; integer programming.

1 Introduction

Knapsack problem is considered as a typical problem in combinatorial optimisation with many real-life applications. For this reason, there are many versions of this problem. For all versions, there is a set of n

**Corresponding author: E-mail: asmaa.sobhy@gmail.com;*

items where $0 \le j \le n$ each item has a weight w_j and a profit p_j. The objective is to select some of the items to be included in a collection so that the total value is maximized and the total weight must not exceed a given capacity. Knapsack problem is NP-complete problem, so it is easy to be qualified, but difficult to be achieved in the case of a large scale.

The MKP is one of the most known versions of knapsack problem. We get MKP when the items should be chosen from different classes and several knapsack should be filled. MKP is used in many scheduling and loading problems in operations research.

The studying of the knapsack problem is back to 1879 [1]. In 1980 Gallo et al. [2] introduced the quadratic knapsack problem (QKP) for first time. Recently the QKP with multiple constraints was studied by Wang et al. [3]. Multiple choice knapsack problem is solved by Dyer et al. [4] using a branch and bound algorithm. Fujimoto and Yamada [5] introduced an exact algorithm for the knapsack sharing problem. For a class of knapsack problems with binary weights, Greedy algorithms are used by Gorski et al. [6]. Also, McLay and Jacobson [7] introduced Algorithms for the bounded set-up knapsack problem. Zhang [8] introduce a Comparative study of several intelligent algorithms for KP. Exact Solution Algorithms for Multi-dimensional Multiple choice Knapsack Problems was presented by Farhad et al. [9]. One level reformulation of the bilevel Knapsack problem using dynamic programming was introduced by Brotcorne et al. [10]. Qiu, and Kern [11] Presented Improved approximation algorithms for a bilevel knapsack problem.

For the above literature, the knapsack problems were studied in a deterministic environment, since the weights and the values have positive crisp values. However, in the real world the data for the problems are not certain so it is suitable to employ uncertain models and methods to study the KP. Researchers like Schilling [12] used probability theory to represent the KP. Two growth models for multiclass stochastic knapsack problem are presented by Lee and Oh [13]. Beier and Vocking [14] proposed random knapsack in expected polynomial time. Kosuch and Lisser [15] presented stochastic knapsack problems as two-stage. In 2017 Yann et al. [16] introduce a knapsack of unknown capacity.

Also fuzzy theory has important contribution by many researchers to deal with KP. A fuzzy multiple choice knapsack problem was presented by Okada and Gen [17]. Abboud et al. [18] presented an approach that treating the multiobjective multidimensional 0-1 knapsack problem under fuzziness. Kasperski, and Kulej [19] proposed the 0-1 knapsack problem with fuzzy data. For MKP Martello and Toth [20] introduce branch and bound algorithm for 0-1 multiple knapsack problem. Also, Abass [21] presented stability of multiple knapsack problems under fuzziness.

Interval programming is one of the famous methods to model the uncertainty. For interval programming, the bounds of the uncertain coefficients are only required. Inuiguchi and Kume [22] introduced Goal Programming Problems with interval coefficients. A satisfactory solution for interval number linear programming was presented by Liu and Da [23]. Olivera and Antunes [24] surveyed all the methodological aspects of interval programming studied in the past. Jiang et al. [25] introduced a method to solve nonlinear interval number programming by using the interval programming.

The organization of this paper is as follows: the formulation of 0-1 MKP with interval capacities is introduced in section 2. The treatment procedure for interval 0-1 MKP is described in section 3. Also in the same section, a qualitative analysis of some basic stability notions is presented. Section 4 is devoted to determining the stability set of first kind. A numerical example (case study) is introduced in section 5. Finally, the conclusions are presented in section 6.

2 Problem Formulation

For MKP, we have a set of items *n*, should be filled in *m* knapsack of interval capacities $\tilde{c}_i$, $i = 1,...,m$. Each item has a weight w_j and a value p_j, where $0 \le j \le n$.

First we will define the following necessary notations:

Parameters

$\tilde{c}_i = \left[c_i^L, c_i^R\right]$ is interval capacities, $i = 1,...,m$.
n : number of items j , $j = 1,...,n$.
m : number of knapsack i
p_j : profit of each item j .
w_j : weight of each item j .

Variables

$$x_{ij} = \begin{cases} 1 & \text{if item } j \text{ is assigned to knabsack } i \\ 0 & \text{otherwise} \end{cases}$$

2.1 Optimization model

The MKP with interval capacities is as follows:

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} p_j x_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{n} w_j x_{ij} \le \left[c_i^L, c_i^R\right], \qquad i = 1,...,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} \le 1, \qquad j = 1,...,n \tag{3}$$

$$x_{ij} \in \{0,1\}, \qquad i = 1,...,m,\ j = 1,...,n \tag{4}$$

Definition: The possibility degree of interval capacity represents the certain degree that interval capacity is larger than total weight. So for interval capacity $\tilde{c}_i = \left[c_i^L, c_i^R\right]$ and total weight $\sum_{j=1}^{n} w_j$, the possibility degree can be defined as follows:

$$P_{\left(\tilde{c}_i \ge \sum_{j=1}^{n} w_j\right)} = \begin{cases} 0 & \sum_{j=1}^{n} w_j > c_i^R \\ \dfrac{c_i^R - \sum_{j=1}^{n} w_j}{c_i^R - c_i^L} & c_i^L < \sum_{j=1}^{n} w_j \le c_i^R \\ 1 & \sum_{j=1}^{n} w_j \le c_i^L \end{cases}$$

So $P_{\left(\tilde{c}_i \geq \sum_{j=1}^{n} w_j\right)} \geq \lambda_i$ is the possibility degree of the *i*th constraint where $0 \leq \lambda_i \leq 1$, $i = 1, 2, ..., m$ is a predetermined possibility degree level.

3 The Treatment Procedure for the Interval 0-1 MKP

We can apply the possibility degree on interval capacity constraints as follow:

$$P_{\tilde{c}_i \geq \sum_{j=1}^{n} w_j x_{ij}} \geq \lambda_i$$

where

$$0 \leq \lambda_i \leq 1 \qquad i = 1, ..., m$$

According to the definition, we have:

$$\frac{c_i^R - \sum_{j=1}^{n} w_j x_{ij}}{c_i^R - c_i^L} \geq \lambda_i \qquad i = 1, ..., m$$

$$0 \leq \lambda_i \leq 1$$

So the deterministic form of 0-1 MKP will take the following form:

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} p_j x_{ij} \tag{5}$$

subject to

$$\frac{c_i^R - \sum_{j=1}^{n} w_j x_{ij}}{c_i^R - c_i^L} \geq \lambda_i \qquad i = 1, ..., m \tag{6}$$

$$\sum_{i=1}^{m} x_{ij} \leq 1, \qquad j = 1, ..., n \tag{7}$$

$$0 \leq \lambda_i \leq 1 \tag{8}$$

$$x_{ij} \in \{0, 1\}, \qquad i = 1, ..., m, \; j = 1, ..., n \tag{9}$$

a point $\left(x_{ij}^{*}\right)$ which satisfies set of constraints (6)-(9) is said to be optimal solution of problem (5)-(9), if and only if there does not exist another $\left(x_{ij}\right)$ which satisfy set of constraints (6)-(9) such that $\sum_{i=1}^{m}\sum_{j=1}^{n} p_j x_{ij} \le \sum_{i=1}^{m}\sum_{j=1}^{n} p_j x_{ij}^{*}$

3.1 Qualitative analysis of some basic stability notions

In this section a parametric influence on possibility degree level λ_i is presented.

Let $\lambda_i, i = 1,...,m$ are assumed to be parameters rather than constants. The decision space of the problem (5)-(9) can be defined as follows:

$$G(\lambda) = \left\{\left(x_{ij}, c_i\right) \in R^{m(n+1)} \text{which satisfy the set of constraints } (6)-(9)\right\}.$$

We introduce the definitions of some basic stability notions for the problem (5)-(9) are given in the following. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind [26, 27].

3.2 The set of feasible parameters

The set of feasible parameters of problem (5)-(9) which is denoted by *A* is defined by:

$$U = \left\{(\lambda) \in R^{I} \,\middle|\, G(\lambda) \neq \phi\right\}.$$

3.3 The solvability set

The solvability set of problem (5)-(9) which is denoted by *V*, is defined by:

$$V = \left\{(\lambda) \in U \,\middle|\, \text{problem}(5)-(9) \text{ has optimal solutions}\right\}.$$

3.4 The stability set of first kind

Suppose that $\lambda^{*} \in V$ with a corresponding optimal solution x_{ij}^{*} for problem (5)-(9) together with optimal parameters c_i^{*}. The stability set of the first kind of problem (5)-(9) that is denoted by $S\left(x_{ij}^{*}\right)$ is defined by:

$$S\left(x_{ij}^{*}\right) = \left\{(\lambda) \in V \,\middle|\, \begin{array}{l} x_{ij}^{*}, i = 1,2,...,m,\ j = 1,2,...,n \text{ is optimal solution of problem} \\ (5)-(9) \text{ with the corresponding optimal parameters } c_i^{*} \end{array}\right\}.$$

4 Determination of the Stability Set of First Kind

The Lagrange function of problem (5)-(9) can be written as follows:

$$L = \sum_{i=1}^{m}\sum_{j=1}^{n} p_j x_{ij} + \theta_i\left(\sum_{j=1}^{n} w_j x_{ij} + \lambda_i\left(c_i^{R} - c_i^{L}\right) - c_i^{R}\right) + \beta_j\left(\sum_{i=1}^{m} x_{ij} - 1\right) + \delta_{ij}\left(-x_{ij}\right) + \alpha_{ij}\left(x_{ij} - 1\right)$$

The Kuhn-Tucker necessary optimality conditions corresponding to the problem (5)-(9) will take the following form:

$$\frac{\partial L}{\partial x_{ij}} = 0 \ , \quad i = 1,\dots,m \ , \ j = 1,\dots,n$$

$$\frac{c_i^R - \sum_{j=1}^{n} w_j x_{ij}}{c_i^R - c_i^L} \ge \lambda_i \qquad i = 1,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} \le 1, \qquad j = 1,\dots,n$$

$$x_{ij} \in \{0,1\}, \qquad i = 1,\dots,m, \ j = 1,\dots,n$$

$$\theta_i \left(\sum_{j=1}^{n} w_j x_{ij} + \lambda_i \left(c_i^R - c_i^L\right) - c_i^R \right) = 0$$

$$\beta_j \left(\sum_{i=1}^{m} x_{ij} - 1 \right) = 0$$

$$\delta_{ij} \left(-x_{ij}\right) = 0$$

$$\alpha_{ij} \left(x_{ij} - 1\right) = 0$$

$$\theta_i, \beta_j, \delta_{ij}, \alpha_{ij} \ge 0, \qquad \forall i, j$$

Where $\theta_i, \beta_j, \delta_{ij}$ and α_{ij} are the Lagrange multipliers and all the relations of the above system are evaluated at the optimal solution x_{ij}^*. According to whether any of the variables $\theta_i, \beta_j, \delta_{ij}$ and α_{ij} are zero or positive, then the stability set of the first kind $S\left(x_{ij}^*\right)$ can be determined.

5 Numerical Example (Case Study)

To demonstrate the solution method for the MKP with interval capacities we consider the following case study. In a wharf of an Egyptian port for one day, the number of items *n* = 15 and the number of knapsack *m* = 10. The following table contains the values of

1 - Profit of each item j p_j.

2 - Weight of each item j w_j.

3 - Interval capacities $\tilde{c}_i$

where $i = 1,\dots,m, \ j = 1,\dots,n$

Table 1.

p_j	w_j	$\tilde{c}_i$
3	5	[1,3]
5	7	[2,4]
2	4	[5,7]
7	9	[3,5]
4	6	[6,8]
6	8	[4,6]
3	5	[10,12]
4	6	[9,11]

5	7	[8,10]
1	2	[7,9]
6	8	
8	10	
4	6	
2	4	
3	5	

Let $\lambda_i = 0.5 \ \forall \ i = 1,...,10$ then we get the deterministic form of MKP for this example as following:

$$\max \left(\begin{array}{l}
3x_{11} + 5x_{12} + 2x_{13} + 7x_{14} + 4x_{15} + 6x_{16} + 3x_{17} + 4x_{18} + 5x_{19} + x_{1(10)} + 6x_{1(11)} + 8x_{1(12)} + 4x_{1(13)} + 2x_{1(14)} + 3x_{1(15)} + \\
3x_{21} + 5x_{22} + 2x_{23} + 7x_{24} + 4x_{25} + 6x_{26} + 3x_{27} + 4x_{28} + 5x_{29} + x_{2(10)} + 6x_{2(11)} + 8x_{2(12)} + 4x_{2(13)} + 2x_{2(14)} + 3x_{2(15)} + \\
3x_{31} + 5x_{32} + 2x_{33} + 7x_{34} + 4x_{35} + 6x_{36} + 3x_{37} + 4x_{38} + 5x_{39} + x_{3(10)} + 6x_{3(11)} + 8x_{3(12)} + 4x_{3(13)} + 2x_{3(14)} + 3x_{3(15)} + \\
3x_{41} + 5x_{42} + 2x_{43} + 7x_{44} + 4x_{45} + 6x_{46} + 3x_{47} + 4x_{48} + 5x_{49} + x_{4(10)} + 6x_{4(11)} + 8x_{4(12)} + 4x_{4(13)} + 2x_{4(14)} + 3x_{4(15)} + \\
3x_{51} + 5x_{52} + 2x_{53} + 7x_{54} + 4x_{55} + 6x_{56} + 3x_{57} + 4x_{58} + 5x_{59} + x_{5(10)} + 6x_{5(11)} + 8x_{5(12)} + 4x_{5(13)} + 2x_{5(14)} + 3x_{5(15)} + \\
3x_{61} + 5x_{62} + 2x_{63} + 7x_{64} + 4x_{65} + 6x_{66} + 3x_{67} + 4x_{68} + 5x_{69} + x_{6(10)} + 6x_{6(11)} + 8x_{6(12)} + 4x_{6(13)} + 2x_{6(14)} + 3x_{6(15)} + \\
3x_{71} + 5x_{72} + 2x_{73} + 7x_{74} + 4x_{75} + 6x_{76} + 3x_{77} + 4x_{78} + 5x_{79} + x_{7(10)} + 6x_{7(11)} + 8x_{7(12)} + 4x_{7(13)} + 2x_{7(14)} + 3x_{7(15)} + \\
3x_{81} + 5x_{82} + 2x_{83} + 7x_{84} + 4x_{85} + 6x_{86} + 3x_{87} + 4x_{88} + 5x_{89} + x_{8(10)} + 6x_{8(11)} + 8x_{8(12)} + 4x_{8(13)} + 2x_{8(14)} + 3x_{8(15)} + \\
3x_{91} + 5x_{92} + 2x_{93} + 7x_{94} + 4x_{95} + 6x_{96} + 3x_{97} + 4x_{98} + 5x_{99} + x_{9(10)} + 6x_{9(11)} + 8x_{9(12)} + 4x_{9(13)} + 2x_{9(14)} + 3x_{9(15)} + \\
3x_{(10)1} + 5x_{(10)2} + 2x_{(10)3} + 7x_{(10)4} + 4x_{(10)5} + 6x_{(10)6} + 3x_{(10)7} + 4x_{(10)8} + 5x_{(10)9} + x_{10(10)} + 6x_{10(11)} + 8x_{10(12)} + 4x_{10(13)} \\
+2x_{10(14)} + 3x_{10(15)}
\end{array} \right)$$

Subject to

$$5x_{11} + 7x_{12} + 4x_{13} + 9x_{14} + 6x_{15} + 8x_{16} + 5x_{17} + 6x_{18} + 7x_{19} + 2x_{1(10)} + 8x_{1(11)} + \\ 10x_{1(12)} + 6x_{1(13)} + 4x_{1(14)} + 5x_{1(15)} \leq 2$$

$$5x_{21} + 7x_{22} + 4x_{23} + 9x_{24} + 6x_{25} + 8x_{26} + 5x_{27} + 6x_{28} + 7x_{29} + 2x_{2(10)} + 8x_{2(11)} + \\ 10x_{2(12)} + 6x_{2(13)} + 4x_{2(14)} + 5x_{2(15)} \leq 3$$

$$5x_{31} + 7x_{32} + 4x_{33} + 9x_{34} + 6x_{35} + 8x_{36} + 5x_{37} + 6x_{38} + 7x_{39} + 2x_{3(10)} + 8x_{3(11)} + \\ 10x_{3(12)} + 6x_{3(13)} + 4x_{3(14)} + 5x_{3(15)} \leq 6$$

$$5x_{41} + 7x_{42} + 4x_{43} + 9x_{44} + 6x_{45} + 8x_{46} + 5x_{47} + 6x_{48} + 7x_{49} + 2x_{4(10)} + 8x_{4(11)} + \\ 10x_{4(12)} + 6x_{4(13)} + 4x_{4(14)} + 5x_{4(15)} \leq 4$$

$$5x_{51} + 7x_{52} + 4x_{53} + 9x_{54} + 6x_{55} + 8x_{56} + 5x_{57} + 6x_{58} + 7x_{59} + 2x_{5(10)} + 8x_{5(11)} + \\ 10x_{5(12)} + 6x_{5(13)} + 4x_{5(14)} + 5x_{5(15)} \leq 7$$

$$5x_{61} + 7x_{62} + 4x_{63} + 9x_{64} + 6x_{65} + 8x_{66} + 5x_{67} + 6x_{68} + 7x_{69} + 2x_{6(10)} + 8x_{6(11)} + \\ 10x_{6(12)} + 6x_{6(13)} + 4x_{6(14)} + 5x_{6(15)} \leq 5$$

$$5x_{71} + 7x_{72} + 4x_{73} + 9x_{74} + 6x_{75} + 8x_{76} + 5x_{77} + 6x_{78} + 7x_{79} + 2x_{7(10)} + 8x_{7(11)} + 10x_{7(12)} + 6x_{7(13)} + 4x_{7(14)} + 5x_{7(15)} \leq 11$$

$$5x_{81} + 7x_{82} + 4x_{83} + 9x_{84} + 6x_{85} + 8x_{86} + 5x_{87} + 6x_{88} + 7x_{89} + 2x_{8(10)} + 8x_{8(11)} + 10x_{8(12)} + 6x_{8(13)} + 4x_{8(14)} + 5x_{8(15)} \leq 10$$

$$5x_{91} + 7x_{92} + 4x_{93} + 9x_{94} + 6x_{95} + 8x_{96} + 5x_{97} + 6x_{98} + 7x_{99} + 2x_{9(10)} + 8x_{9(11)} + 10x_{9(12)} + 6x_{9(13)} + 4x_{9(14)} + 5x_{9(15)} \leq 9$$

$$5x_{(10)1} + 7x_{32} + 4x_{(10)3} + 9x_{(10)4} + 6x_{(10)5} + 8x_{(10)6} + 5x_{(10)7} + 6x_{(10)8} + 7x_{(10)9} + 2x_{(10)(10)} + 8x_{(10)(11)} + 10x_{(10)(12)} + 6x_{(10)(13)} + 4x_{(10)(14)} + 5x_{(10)(15)} \leq 8$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81} + x_{91} + x_{(10)1} \leq 1$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} + x_{72} + x_{82} + x_{92} + x_{(10)2} \leq 1$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} + x_{83} + x_{93} + x_{(10)3} \leq 1$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} + x_{94} + x_{(10)4} \leq 1$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} + x_{75} + x_{85} + x_{95} + x_{(10)5} \leq 1$$

$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} + x_{76} + x_{86} + x_{96} + x_{(10)6} \leq 1$$

$$x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} + x_{77} + x_{87} + x_{97} + x_{(10)7} \leq 1$$

$$x_{18} + x_{28} + x_{38} + x_{48} + x_{58} + x_{68} + x_{78} + x_{88} + x_{98} + x_{(10)8} \leq 1$$

$$x_{19} + x_{29} + x_{39} + x_{49} + x_{59} + x_{69} + x_{79} + x_{89} + x_{99} + x_{(10)9} \leq 1$$

$$x_{1(10)} + x_{2(10)} + x_{3(10)} + x_{4(10)} + x_{5(10)} + x_{6(10)} + x_{7(10)} + x_{8(10)} + x_{9(10)} + x_{(10)(10)} \leq 1$$

$$x_{1(11)} + x_{2(11)} + x_{3(11)} + x_{4(11)} + x_{5(11)} + x_{6(11)} + x_{7(11)} + x_{8(11)} + x_{9(11)} + x_{(10)(11)} \leq 1$$

$$x_{1(12)} + x_{2(12)} + x_{3(12)} + x_{4(12)} + x_{5(12)} + x_{6(12)} + x_{7(12)} + x_{8(12)} + x_{9(12)} + x_{(10)(12)} \leq 1$$

$$x_{1(13)} + x_{2(13)} + x_{3(13)} + x_{4(13)} + x_{5(13)} + x_{6(13)} + x_{7(13)} + x_{8(13)} + x_{9(13)} + x_{(10)(13)} \leq 1$$

$$x_{1(14)} + x_{2(14)} + x_{3(14)} + x_{4(14)} + x_{5(14)} + x_{6(14)} + x_{7(14)} + x_{8(14)} + x_{9(14)} + x_{(10)(14)} \leq 1$$

$$x_{1(15)} + x_{2(15)} + x_{3(15)} + x_{4(15)} + x_{5(15)} + x_{6(15)} + x_{7(15)} + x_{8(15)} + x_{9(15)} + x_{(10)(15)} \leq 1$$

$$x_{ij} \in \{0,1\}, \qquad i = 1,...,10,\ j = 1,...,15$$

By solving the above problem the optimal solution of the above problem can be obtained: $x^*_{1(10)} = x^*_{35} = x^*_{4(14)} = x^*_{61} = x^*_{72} = x^*_{73} = x^*_{8(12)} = x^*_{94} = x^*_{(10)6} = 1$ and all other variables = 0.

and objective function value = 43.
These results means that only items 1, 2, 3, 4, 5, 6, 10, 12 and 14 are assigned to knabsacks 6, 7, 7, 9, 3, 10, 1, 8 and 4 respectively with maximum total value is 43.

For the parametric study, the set of feasible parameters, solvability set and the stability set of the first kind are calculated as follows:

Set of feasible parameters:

$$U = \{(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7,\lambda_8,\lambda_9,\lambda_{10}) \in R \mid 0 \le \lambda_1,...,\lambda_{10} \le 1\}.$$

Solvability set:

$$V = \{(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7,\lambda_8,\lambda_9,\lambda_{10}) \in U \mid 0 \le \lambda_1,...,\lambda_{10} \le 1\}.$$

Stability set of the first kind:

$$S\left(x^*_{ij}, i = 1,\dots,10, j = 1,\dots 15\right) = \left\{ \begin{array}{l} (\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7,\lambda_8,\lambda_9,\lambda_{10}) \in V \\ \left| \lambda_1 = \lambda_6 = \lambda_7 = \lambda_8 = \lambda_9, \lambda_{10} = \frac{1}{2},\ 0 \le \lambda_2,\lambda_3,\lambda_4,\lambda_5 \le 1 \right. \end{array} \right\}.$$

6 Conclusion

In this paper, multiple knapsack problems with interval capacities were solved. The concept of stability for MKP with interval capacities is discussed. Some basic stability notions are introduced. The stability set of the first kind for our problem is already determinedu. The solution for the nonlinear knapsack problem with interval capacities would be future topics.

Competing Interests

Authors have declared that no competing interests exist.

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